UNIT- 5 – ANALYTICAL GEOMENTRY

I. Answer the Following (2 marks)

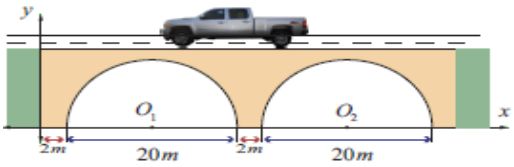
1. Determine whether $x+ y - 1 = 0$ is the equation of a diameter of the circle $x^2 + y^2 -6x +4y + c = 0$ for all possible values of *c.*
2. *If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.*
3. Obtain the equation of the circles with radius 5 cm and touching *x*-axis at the origin in general form.
4. If $y = 2\sqrt{2}x + c$ is a tangent to the circle $x^2 + y^2 = 16$, find the value of c.
5. If the equation $3x^2 + (3 – p) xy + qy^2 – 2px = 8$ pq represents a circle, find *p* and *q*. Also, determine the centre and radius of the circle.
6. Find the length of Latus rectum of the parabola $y^2 = 4ax$.
7. Find the equation of the parabola in each of the cases given below: Vertex (1, -2) and focus (4, - 2).
8. Find the equation of the hyperbola in each of the cases given below: foci (±2, 0), eccentricity = 3/2.
9. Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $y^2 = 16x$.
10. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following: $\frac{y^2}{16} - \frac{x^2}{9} = 1$.
11. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.
12. An equation of the elliptical part of an optical lens system is $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The parabolic part of the system has a focus in common with the right focus of the ellipse .The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of the parabola.

II. Answer the Following (3 Marks)

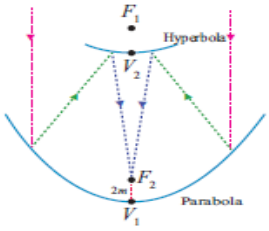
1. A line $3x + 4y + 10 = 0$ cuts a chord of length 6 units on a circle with centre of the circle (2, 1). Find the equation of the circle in general form.
2. Find the equation of the circle through the points (1, 0), (-1, 0), and (0, 1).
3. Find the length of Latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
4. Identify the type of conic section for each of the equations.
   i) $11x^2 – 25y^2 – 44x + 50y – 256 = 0$. Ii) $y^2 + 4x + 3y + 4 = 0$.
5. Find the equations of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at (1, 3).
6. Find the equations of the two tangents that can be drawn from (5, 2) to the ellipse $2x^2 + 7y^2 = 14$.
7. Prove that the point of intersection of the tangents at ' t_1' and 't_2' on the parabola $y^2 = 4ax$ is$[at_1t_2, a(t_1 + t2)]$.
8. A semielliptical archway over a one-way road has a height of 3*m* and a width of 12*m*. The truck has a width of 3*m* and a height of 27.*m*. Will the truck clear the opening of the archway?
9. At a water fountain, water attains a maximum height of 4*m* at horizontal distance of 0 5. *m* from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 075.*m* from the point of origin.
10. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4*m* when it is 6*m* away from the point of projection. Finally it reaches the ground 12*m* away from the starting point. Find the angle of projection.
11. Points *A* and *B* are 10*km* apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 *km* closer to *A* than *B* . Show that the location of the explosion is restricted to a particular curve and find an equation of it.

III. Answer the Following (5 Marks)

1. Find the equation of the circle passing through the points (1, 1), (2, -1), and (3, 2).
2. A road bridge over an irrigation canal have two semi circular vents each with a span of 20*m* and the supporting pillars of width 2*m*. Use Fig.5.16 to write the equations that represent the semi-verticular vents.

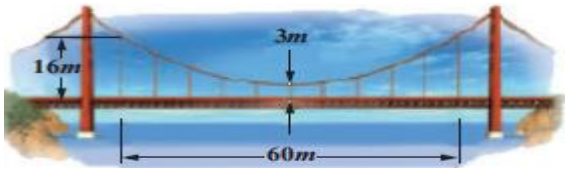


3. Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $y^2 - 4y - 8x + 12 = 0$.
4. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following: $9x^2 - y^2 - 36x - 6y + 18 = 0$.
5. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.
6. If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.
7. Two coast guard stations are located 600 km apart at points *A*(0, 0)and *B*(0, 600) . A distress signal from a ship at *P* is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station *A* than it is from station *B*. Determine the equation of hyperbola that passes through the location of the ship.
8. Certain telescopes contain both parabolic mirror and a hyperbolic mirror. In the telescope shown in figure 5.68 the parabola and hyperbola share focus F_1, *which* is 14*m* above the vertex of the parabola. The hyperbola's second focus F_2 is 2*m* above the parabola's vertex. The vertex of the hyperbolic mirror is 1*m* below F_1. Position a coordinate system with the origin at the centre of the hyperbola and with the foci on the *y* -axis. Then find the equation of the hyperbola.



9. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16*m*, and the height at the edge of the road must be sufficient for a truck 4*m* high to clear if the highest point of the opening is to be 5*m* approximately. How wide must the opening be?

10. Parabolic cable of a 60*m* portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6*m* along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



11. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150*m* tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.

12. A rod of length 1 2. *m* moves with its ends always touching the coordinate axes. The locus of a point *P* on the rod, which is 0 3. *m* from the end in contact with *x* -axis is an ellipse. Find the eccentricity.
13. Assume that water issuing from the end of a horizontal pipe, 7 5. *m* above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2 5. *m* below the line of the pipe, the flow of water has curved outward 3*m* beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?