UNIT- 6 – VECTOR ALGEBRA

I. Answer the Following (2 marks)

1. With usual notations, in any triangle *ABC*, prove the following by vector method $c^2 = a^2 + b^2 - 2$ ab cos C.
2. A particle acted upon by constant forces $2\hat{\imath} + 5\hat{\jmath} + 6\hat{k}$ *and* $-\hat{\imath} - 2\hat{\jmath} - \hat{k}$ is displaces from the point (4, -3, -2) to the point (6, 1, -3). Find the total work done by the forces.
3. Prove by vector method that an angle in a semi-circle is a right angle.
4. Show that the four points lie on a same plane (6,-7,0), (16,-19,-4), (0,3,-6), (2,-5,10) lie on a same plane.
5. The volume of the parallelepiped whose coterminus edges are $7\hat{\imath} + \lambda\hat{\jmath} - 3\hat{k}, \hat{\imath} + 2\hat{\jmath} - \hat{k}, -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$ is 90 cubic units. Find the value of λ.
6. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors represented by concurrent edges of a parallelepiped of volume 4 cubic units, find the value of $(\vec{a} + \vec{b}).(\vec{b}\ x\ \vec{c}) + (\vec{b} + \vec{c}).(\vec{c}\ x\ \vec{a}) + (\vec{c} + \vec{a}).(\vec{a}\ x\ \vec{b})$.
7. Let $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_2\hat{k}$. If $c_1 = 1$ and $c_2 = 2$, find c_3 such that $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar.
8. Prove that $[\vec{a}\ x\ \vec{b}, \vec{b}\ x\ \vec{c}, \vec{c}\ x\ \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$.
9. Show that the points (2,3,4), (-1, 4,5) and (8,1,2) are collinear.
10. Find a parametric form of vector equation of a plane which is at a distance of 7 units from the origin having 3,-4,5 as direction ratios of a normal to it.
11. Find the vector and Cartesian equations of the plane passing through the point with position vector $2\vec{\imath} + 6\vec{\jmath} + 3\vec{k}$ and normal to the vector $\vec{\imath} + 3\vec{\jmath} + 5\vec{k}$.
12. Find the angle between the planes $\vec{r}.(\hat{\imath} + \hat{\jmath} - 2\hat{k})$=3 and 2x-2y+2z = 2.
13. Find the length of the perpendicular from the point (1, -2, 3) to plane x-y+z = 5.

II. Answer the Following (3 Marks)

1. Forces of magnitudes $5\sqrt{2}$ *and* $10\sqrt{2}$ units acting in the directions $3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $10\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$ to the point with position vector $6\hat{\imath} + \hat{\jmath} - 3\hat{k}$. Find the work done by the forces.

2. Find the altitude of a parallelepiped determined by the vectors $\vec{a} = -2\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$, $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ and $\vec{c} = -3\vec{\imath} + \vec{\jmath} + 4\vec{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ *and* $\vec{c}$.

3. If the vectors $a\hat{\imath} + a\hat{\jmath} + c\hat{k}$, $\hat{\imath} + \hat{k}$ and $c\hat{\imath} + c\hat{\jmath} + b\hat{k}$ are coplanar, prove that c is the geometric mean of *a and b.*

4. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ *and* $\vec{b}$. If the angle between $\vec{a}$ *and* $\vec{b}$ is $\frac{\pi}{6}$, show that $[\vec{a}, \vec{b}, \vec{c}]^2 = \frac{1}{4}|\vec{a}|^2\,|\vec{b}|^2$.

5. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{c} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{a}x(\vec{b}x\vec{c}) = l\vec{a} + m\vec{b} + n\vec{c}$, find the values of l, m, n.

6. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non – parallel and $\hat{a}\ x\ (\hat{b}\ x\ \hat{c}) = \frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\vec{c}$.

7. If the straight line joining the points (2,1,4) and (a-1,4,-1) is parallel to the line joining the points (0, 2, b-1) and (5,3,-2), find the values of a and b.

8. If a plane meets the coordinate axes at A,B,C such that the centriod of the triangle ABC is the point (u,v,w), find the equation of the plane.

9. If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of *m*.

10. Find the equation of the plane passing through the line of intersection of the planes x+2y+3z =2 and x-y+z+11=3, and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, -1).

11. Find the equation of the plane which passes through the point (3, 4, -1) and is parallel to the plane 2x-3y+5z+7 = 0. Also, find the distance between the two planes.

III. <u>Answer the Following (5 Marks)</u>

1. Prove by vector method that sin(α – β) = sinα cosβ – cosα sinβ.
2. Using vector method, prove that cos (α - β) = cosα cosβ + sinα sinβ.
3. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$, $\vec{c} = -\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, verify that

   (i) $\left(\vec{a}\, x\, \vec{b}\right) x\, \vec{c} = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\vec{a}$ (ii) $\vec{a}\, x \left(\vec{b}\, x\, \vec{c}\right) = (\vec{a}\,.\vec{c})\vec{b} - (\vec{a}\,.\vec{b})\vec{c}$
4. Find the points where the straight line passes through (6, 7, 4) and (8, 4, 9) cuts the *xz and yz* planes.
5. If the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-m}{2} = z$ intersect at a point, find the value of m.
6. Find the foot of the perpendicular drawn from the point (5, 4, 2).
7. Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane 2x+6y+6z = 9.
8. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points (3, 6 ,-2), (-1, -2, 6) and (6, -4, -2).
9. Show that the straight lines $\vec{r} = (5\hat{\imath} + 7\hat{\jmath} - 3\hat{k})$+ s$(4\hat{\imath} + 4\hat{\jmath} - 5\hat{k})$ and $\vec{r} = (8\hat{\imath} + 4\hat{\jmath} + 5\hat{k})$ + t $(7\hat{\imath} + \hat{\jmath} + 3\hat{k})$ are coplanar. Find the vector equation of the plane in which they lie.
10. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(2\hat{\imath} - 7\hat{\jmath} + 4\hat{k})$=3 and 3x-5y+4z+11=0, and the point (-2, 1, 3).
11. Find the point of intersection of the line x-1= $\frac{y}{2}$ = z+1 with the plane 2x-y+2z = 2. Also, find the angle between the line and the plane.
12. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4, 3, 2) to the plane x+2y+3z = 2.