

UNIT- 6 – ANALYTICAL GEOMETRY

I. Answer the Following (2 marks)

1. Find the locus of a point P moves such that its distances from two fixed points A (1, 0) and B (5, 0) , are always equal.
2. Find the locus of P, if for all values of α , the co-ordinates of a moving point P is
(i) $(9\cos \alpha, 9 \sin \alpha)$ (ii) $(9\cos \alpha, 6 \sin \alpha)$.
3. If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $x = a \cos^3 \theta$, $y = a \sin^3 \theta$.
4. The length of the perpendicular drawn from the origin to a line is 12 and makes an angle 150° with positive direction of the x-axis. Find the equation of the line.
5. Area of the triangle formed by a line with the coordinate axes, is 36 square units. Find the equation of the line if the perpendicular drawn from the origin to the line makes an angle of 45° with positive the x-axis.
6. If P (r, c) is mid-point of a line segment between the axes, then show that $\frac{x}{r} + \frac{y}{c} = 2$.
7. Find the equation of the line, if the perpendicular drawn from the origin makes an angle 30° with x-axis and its length is 12.
8. A straight line is passing through the point A(1, 2) with slope $\frac{5}{12}$. Find points on the line which are 13 units away from A.
9. A 150m long train is moving with constant velocity of 12.5 m/s. Find (i) the equation of the motion of the train, (ii) time taken to cross a pole. (iii) The time taken to cross the bridge of length 850m is?
10. Find the nearest point on the line $2x + y = 5$ from the origin.
11. Show that the lines are $3x + 2y + 9=0$ and $12x + 8y - 15 =0$ are parallel lines.
12. Find the distance between the line $4x + 3y + 4 = 0$, and a point (i) (-2, 4) (ii) (7, -3).
13. If p_1 and p_2 are the lengths of the perpendiculars from the origin to the straight lines $x \sec \theta + y \operatorname{cosec} \theta = 2a$ and $x \cos \theta - y \sin \theta = a \cos 2 \theta$, then prove that $p_1^2 + p_2^2 = a^2$.
14. Find the distance between the parallel lines
(i) $12x + 5y = 7$ and $12x + 5y + 7= 0$ (ii) $3x - 4y + 5=0$ and $6x - 8y - 15 = 0$.
15. Find the image of the point (-2, 3) about the line $x + 2y - 9 =0$.
16. A photocopy store charges ₹ 1.50 per copy for the first 10 copies and ₹ 1.00 per copy after the 10th copy. Let x be the number of copies, and let y be the total cost of photocopying. (i) Draw graph of the cost as x goes from 0 to 50 copies. (ii) Find the cost of making 40 copies.
17. Find the combined equation of the straight lines whose separate equations are $x - 2y - 3 =0$ and $x + y + 5 = 0$.

II. Answer the Following (3 marks)

1. A straight rod of length 8 units slides with its ends A and B always on the x and y axes respectively. Find the locus of the mid-point of the line segment AB.
2. Find the equation of the locus of a point such that the sum of the squares of the distance from the points (3, 5), (1, -1) is equal to 20.
3. If O is origin and R is a variable point on $y^2 = 4x$, then find the equation of the locus of the mid-point of the line segment OR.
4. The coordinates of a moving point P are $\left(\frac{a}{2} (\operatorname{cosec}\theta + \sin\theta), \frac{b}{2} (\operatorname{cosec}\theta - \sin\theta)\right)$, where θ is a variable parameter. Show that the equation of the locus P is $b^2 x^2 - a^2 y^2 = a^2 b^2$.
5. The seventh term of an arithmetic progression is 30 and tenth term is 21.
 - (i) Find the first three terms of an A.P.
 - (ii) Which term of the A.P. is zero (if exists)
 - (iii) Find the relationship between Slope of the straight line and common difference of A.P.
6. Express the equation $\sqrt{3}x - y + 4 = 0$ in the following equivalent form:
 - (i) Slope and Intercept form
 - (ii) Intercept form
 - (iii) Normal form
7. If p is length of perpendicular from origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.
8. The normal boiling point of water is 100°C or 212°F and the freezing point of water is 0°C or 32°F . (i) Find the linear relationship between C and F Find (ii) the value of C for 98.6°F and (iii) the value of F for 38°C .
9. Population of a city in the years 2005 and 2010 are 1,35,000 and 1,45,000 respectively. Find the approximate population in the year 2015. (assuming that the growth of population is constant)
10. If (-4, 7) is one vertex of a rhombus and if the equation of one diagonal is $5x - y + 7 = 0$, then find the equation of another diagonal.
11. Find the equations of straight lines which are perpendicular to the line $3x + 4y - 6 = 0$ and are at a distance of 4 units from (2,1).
12. Find the equation of a straight line parallel to $2x + 3y = 10$ and which is such that the sum of its intercepts on the axes is 15.

13. If the line joining two points A (2,0) and B (3,1) is rotated about A in anticlockwise direction through an angle of 15° , then find the equation of the line in new position.
14. A ray of light coming from the point (1,2) is reflected at a point A on the x-axis and it passes through the point (5,3). Find the co-ordinates of the point A.
15. Show that $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines.
16. Show that $2x^2 + 3xy - 2y^2 + 3x + y + 1 = 0$ represents a pair of perpendicular lines.
17. Find the equation of the pair of straight lines passing through the point (1, 3) and perpendicular to the lines $2x - 3y + 1 = 0$ and $5x + y - 3 = 0$
18. Find the separate equation of the following pair of straight lines
 - (i) $6(x + 1)^2 + 5(x - 1)(y - 2) - 4(y - 2)^2 = 0$
 - (ii) $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$
19. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is twice that of the other, show that $8h^2 = 9ab$.
20. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.
21. Find p and q, if the following equation represents a pair of perpendicular lines $6x^2 + 5xy - py^2 + 7x + qy - 5 = 0$
22. Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them.
23. Prove that one of the straight lines given by $ax^2 + 2hxy + by^2 = 0$ will bisect the angle between the co-ordinate axes if $(a + b)^2 = 4h^2$
24. Prove that the straight lines joining the origin to the points of intersection of $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$ and $3x - 2y - 1 = 0$ are at right angles.

III. Answer the Following (5 marks)

1. If P (2, -7) is a given point and Q is a point on $2x^2 + 9y^2 = 18$, then find the equations of the locus of the mid-point of PQ.
2. If the points P (6, 2) and Q(-2, 1) and R are the vertices of a ΔPQR and R is the point on the locus $y = x^2 - 3x + 4$, then find the equation of the locus of centroid of ΔPQR
3. If Q is a point on the locus of $x^2 + y^2 + 4x - 3y + 7 = 0$, then find the equation of locus of P which divides segment OQ externally in the ratio 3:4, where O is origin.
4. The sum of the distance of a moving point from the points (4, 0) and (-4, 0) is always 10 units. Find the equation of the locus of the moving point.

5. A spring was hung from a hook in the ceiling. A number of different weights were attached to the spring to make it stretch, and the total length of the spring was measured each time is shown in the following table.

Weight (kg)	2	4	5	8
Length (cm)	3	4	4.5	6

- (i) Draw a graph showing the results.
 - (ii) Find the equation relating the length of the spring to the weight on it.
 - (iii) What is the actual length of the spring.
 - (iv) If the spring has to stretch to 9 cm long, how much weight should be added?
 - (v) How long will the spring be when 6 kilograms of weight on it?
6. A family is using Liquefied petroleum gas (LPG) of weight 14.2 kg for consumption. (Full weight 29.5kg includes the empty cylinders tare weight of 15.3kg.). If it is used with constant rate, then it lasts for 24 days. Then the new cylinder is replaced (i) Find the equation relating the quantity of gas in the cylinder to the days. (ii) Draw the graph for first 96days.
7. Show that the equation $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$.
8. Find the value of k, if the following equation represents a pair of straight lines. Further, find whether these lines are parallel or intersecting, $12x^2 + 7xy - 12y^2 - x + 7y + k = 0$.
9. For what value of k does the equation $12x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represent two straight lines.