UNIT- 9 – APPLICATIONS OF INTEGRATION

I. Answer the Following (2 marks)

1. Evaluate : $\int_0^3 (3x^2 - 4x + 5)dx$.
2. Evaluate: $\int_0^1 \frac{2x+7}{5x^2+9}\,dx$.
3. Evaluate: $\int_0^9 \frac{1}{x+\sqrt{x}}\,dx$.
4. Evaluate : $\int_1^2 \frac{x}{(x+1)(x+2)}\,dx$.
5. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{(1+\sin\theta)(2+\sin\theta)}\,d\theta$.
6. Evaluate : $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\,dx$.
7. Evaluate : $\int_0^{1.5} [x^2]\,dx$, where [x] is the greatest integer function.
8. Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\,dx$.
9. Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$.
10. Evaluate the following definite integrals : $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx$
11. Evaluate the following integrals using properties of integration:

    (i) $\int_{-5}^{5} x\cos\left(\frac{e^x-1}{e^x+1}\right)dx$ (ii) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\,dx$
12. Evaluate : $\int_0^1 e^{-2x}(1 + x - 2x^3)\,dx$.
13. Evaluate the following: $\int_0^1 x^3 e^{-2x}\,dx$
14. Evaluate $\int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^4 x)\,dx$
15. Evaluate $\int_0^1 x^5(1-x^2)^5\,dx$.
16. Evaluate the following: (i) $\int_0^{\frac{\pi}{2}} \sin^{10} x\,dx$ (ii) $\int_0^{\frac{\pi}{2}} \cos^7 x\,dx$
17. Find the area of the region bounded between the curves y = sin x and y = cos x and the lines x = 0 and x= π.
18. Find the area of the region bounded by y = tan x , y = cot x and the lines x = 0,x = $\frac{\pi}{2}$, y = 0.
19. Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$.
20. Find the volume of a sphere of radius a.
21. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = 2x^2$, y = 0 and x = 1.
22. Find, by integration, the volume of the solid generated by revolving about the x-axis, the region enclosed by $y = e^{-2x}$ y = 0, x = 0 and x = 1.

II. Answer the Following (3 Marks)

1. Find an approximate value of $\int_1^{1.5} x^2\,dx$ by applying the right-end rule with the partition {1.1, 1.2, 1.3, 1.4, 1.5}.
2. Find an approximate value of $\int_1^{1.5} (2-x)\,dx$ by applying the mid-point rule with the partition {1.1, 1.2, 1.3, 1.4, 1.5}.
3. Evaluate the following integrals as the limits of sums:

   (i) $\int_0^1 (5x+4)dx$ (ii) $\int_1^2 (4x^2-1)dx$
4. Evaluate: $\int_0^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx$.
5. Show that $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3}\log_e 2$.
6. Prove that $\int_0^{\frac{\pi}{4}} \frac{dx}{a^2\sin^2 x + b^2\cos^2 x} = \frac{1}{ab}\tan^{-1}\left(\frac{a}{b}\right)$, where a, b > 0.
7. Evaluate the following definite integrals: $\int_3^4 \frac{dx}{x^2-4}$.
8. Evaluate the following:

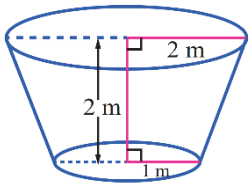
   (i) $\int_0^{2\pi} \sin^7 \frac{x}{4}\,dx$ (ii) $\int_0^{\frac{\pi}{2}} \sin^3\theta \cos^5\theta\,d\theta$ (iii) $\int_0^1 x^2\,(1-x)^3\,dx$
9. Evaluate the following (i) $\int_0^{\infty} x^5 e^{-3x}\,dx$ (ii) $\int_0^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x}\,dx$
10. If $\int_0^{-} e^{-ax^2} x^3\,dx = 32,\ \alpha| > 0, find\ \alpha.$
11. Find the area of the region bounded by 3x − 2y + 6 = 0, x = −3, x =1 and x - axis.
12. Find the area of the region bounded by 2x − y +1 = 0, y = −1, y = 3 and y - axis.
13. Find the area of the region bounded by the curve 2+ x – x^2 + y =0, x - axis, x = −3 and x = 3.
14. Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded between the parabola x = y^2 + 1, the y-axis, and the lines y =1 and y = −1.
15. Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded between the curve y = $\frac{3}{4}\sqrt{x^2-16}$, x ≥ 4, the y-axis, and the lines y =1 and y = 6.

III. Answer the Following (5 Marks)

1. Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{\sin x+\cos x}\,dx$.
2. Prove that $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$.
3. Evaluate the following integrals using properties of integration :

   (i) $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$ (ii) $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$ (iii) $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\,dx$
4. Evaluate the following: (i) $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1} x}\sin^{-1} x}{\sqrt{1-x^2}}\,dx$ (ii) $\int_0^{\frac{\pi}{2}} x^2\cos 2x\,dx$
5. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$.

6. Evaluate the following: (i) $\int_0^{\frac{\pi}{2}} \frac{dx}{1+ 5\cos^2 x}$ (ii) $\int_0^{\frac{\pi}{2}} \frac{dx}{5+ 4\sin^2 x}$
7. Find the area of the region bounded by the line $7x - 5y = 35$, x − axis and the lines $x = -2$ and $x = 3$.
8. Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
9. Find the area of the region bounded by the y -axis and the parabola $x = 5 - 4y - y^2$.
10. Find the area of the region bounded by y =cos x, y =sin x , the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.
11. Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$.
12. Using integration find the area of the region bounded by triangle ABC, whose vertices A, B, and C are (−1,1), (3, 2), and (0,5) respectively.
13. Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.
14. Find the volume of a right-circular cone of base radius r and height h.
15. The region enclosed between the graphs of $y = x$ and $y = x^2$ is denoted by R, Find the volume generated when R is rotated through 360° about x-axis.
16. Find, by integration, the volume of the container which is in the shape of a right circular conical frustum as shown in the Figure



17. A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major-axis 20 cm and minor-axis 10 cm about its major-axis. Find its volume using integration.