

CHAPTER - 10 – DIFFERENTIAL CALCULUS

I. Answer the Following (2 marks)

1. Find the derivatives of the following functions with respect to corresponding independent variables:

(i) $f(x) = x - 3 \sin x$

(ii) $y = \sin x + \cos x$

(iii) $f(x) = x \sin x$

(iv) $y = \sin x^\circ$

(v) $y = \log_{10} x$

2. Differentiate : $y = (x^3 - 1)^{100}$.

3. Find $\frac{dy}{dx}$ if $x^2 + y^2 = 1$.

4. Differentiate $y = x^{\sqrt{x}}$.

5. Find $\frac{dy}{dx}$ if $x = at^2 ; 2at, t \neq 0$.

II. Answer the Following (3 marks)

1. Find the derivatives of the following functions with respect to corresponding independent variables:

(i) $y = e^x \sin x$

(ii) $y = \frac{\tan x}{x}$

(iii) $y = \frac{\sin x}{1 + \cos x}$

(iv) $y = \frac{x}{\sin x + \cos x}$

2. Find the derivative of the function $g(t) = \left(\frac{t-2}{2t+1}\right)^9$

3. Differentiate the following :

(i) $y = (x^2 + 4x + 6)^5$

(ii) $y = \tan 3x$

(iii) $y = \sin(e^x)$

(iv) $f(t) = \sqrt[3]{1 + \tan t}$

(v) $y = (1 + \cos^2 x)^6$

4. Find $\frac{dy}{dx}$ if $\sin y = y \cos 2x$.

5. If $y = \tan^{-1} \left(\frac{1+x}{1-x}\right)$, find y' .

6. Find $\frac{dy}{dx}$ if $x = a(t - \sin t), y = a(1 - \cos t)$.

7. Find the derivatives of the following:

(i) $y = x^{\cos x}$

(ii) $x^y = y^x$

(iii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

8. If $y = e^{\tan^{-1}x}$, show that $(1+x^2) y'' + (2x-1)y' = 0$.

III. Answer the Following (5 marks)

1. Differentiate the following :

(i) $s(t) = \sqrt[4]{\frac{t^3+1}{t^3-1}}$

(ii) $y = \frac{\sin^2 x}{\cos x}$

(iii) $y = \frac{e^{3x}}{1+e^x}$

(iv) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

(v) $y = \sin(\tan(\sqrt{\sin x}))$

(vi) $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

2. Find the derivative of x^x with respect to $x \log x$.
3. Find the derivative of $\tan^{-1}(1+x^2)$ with respect to $x^2 + x + 1$.
4. Find the derivatives of the following
 - (i) $\tan^{-1} \sqrt{\frac{1-\cos x}{1+\cos x}}$
 - (ii) $\tan^{-1} \left(\frac{6x}{1-9x^2} \right)$
 - (iii) $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$
 - (iv) $\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$
5. Find the derivative of $\sin x^2$ with respect to x^2 .
6. Find the derivative of $\sin^{-1} \left(\frac{\sin x}{1+x^2} \right)$ with respect to $\tan^{-1} x$.
7. If $u = \tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$ and $v = \tan^{-1} x$, find $\frac{du}{dv}$.
8. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2) y_2 - 3xy_1 - y = 0$.
9. If $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ then prove that at $\theta = \frac{\pi}{2}, y'' = \frac{1}{a}$.
10. If $y = (\cos^{-1} x)^2$, prove that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$.

