ST. JOSEPH STUDY CENTRE

12th STD: Revision Test - 3 **MATHS** **Time: 3 Hrs / Total Marks: 100**

PART - I

i. All question are compulsory. **[20 x 1 = 20]**

ii. Choose the most appropriate answer the given four alternatives and write the option code and the corresponding answer.

1. If $\sin^{-1} x = 2\sin^{-1} \alpha$ has a solution, then ________.

   a. $|\alpha| \leq \frac{1}{\sqrt{2}}$ b. $|\alpha| \geq \frac{1}{\sqrt{2}}$ c. $|\alpha| < \frac{1}{\sqrt{2}}$ d. $|\alpha| > \frac{1}{\sqrt{2}}$

2. The value of $\cos^{-1}(-1) + \tan^{-1}(\infty) + \sin^{-1} 1$ =_________.

   a. $-\pi$ b. $\frac{3\pi}{2}$ c. 30^{o} d. 2π

3. The area between $y^2 = 4x$ and its latus rectum is ________.

   a. $\frac{2}{3}$ b. $\frac{4}{3}$ c. $\frac{8}{3}$ d. $\frac{5}{3}$

4. $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$ if ____________.

   a. $f(2a - x) = f(x)$ b. $f(a - x) = f(x)$ c. $f(x) = - f(x)$ d. $f(-x) = f(x)$

5. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is _________.

   a. $xy = k$ b. $y = k \log x$ c. $y = kx$ d. $\log y = kx$

6. P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then _______.

   a. $P = ce^{kt}$ b. $P = ce^{-kt}$ c. $P = ckt$ d. $Pt = c$

7. If P{X = 0} = 1 – P{X =1}. If E[X] = 3 Var (X), then P{X = 0}.

   a. $\frac{2}{3}$ b. $\frac{2}{5}$ c. $\frac{1}{5}$ d. $\frac{1}{3}$

8. If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

   a. 0 and 12 b. 5 and 17 c. 7 and 19 d. 16 and 24

9. The equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has _______.

   a. no solution b. unique solution
   c. two solutions d. infinite number of solution

10. $\sin^{-1}\left(3\frac{x}{2}\right) + \cos^{-1}\left(3\frac{x}{2}\right)$ = _________.

    a. $\frac{3\pi}{2}$ b. 6x c. 3x d. $\frac{\pi}{2}$

11. The value of $\int_0^1 (\text{siin}^{-1} x)^2\, dx$ is _________.

    a. $\frac{\pi^2}{4} - 1$ b. $\frac{\pi^2}{4} + 2$ c. $\frac{\pi^2}{4} + 1$ d. $\frac{\pi^2}{4} - 2$

12. The integrating factor of the differential equation $\frac{dy}{dx} + p(x)y = Q(x)$ is x, then P(x) = ______.

    a. x b. $\frac{x^2}{2}$ c. $\frac{1}{x}$ d. $\frac{1}{x^2}$

13. The number of arbitrary constants in the general solutions of order n and n + 1 are respectively ________.

a. n – 1, n b. n, n + 1 c. n + 1 , n + 2 d. n + 1, n

14. If $\cot^{-1} 2$ and $\cot^{-1} 3$ are two angles of a triangle, then the third angle is ________.

a. $\frac{\pi}{4}$ b. $\frac{3\pi}{4}$ c. $\frac{\pi}{6}$ d. $\frac{\pi}{3}$

15. $\sec^{-1}\left(\frac{2}{3}\right) + \text{co}\sec^{-1}\left(\frac{2}{3}\right)$ = _______.

a. $\frac{-\pi}{2}$ b. $\frac{\pi}{2}$ c. π d. – π

16. The value of $\int_{-1}^{2}|x|\,dx$ is ________.

a. $\frac{1}{2}$ b. $\frac{3}{2}$ c. $\frac{5}{2}$ d. $\frac{7}{2}$

17. $\int_0^{2a} f(x)\,dx = 0$ if ________.

a. f(2a – x) = f(x) b. f(2a – x) = - f(x) c. f(x) = - f(x) d. f(-x) = f(x)

18. The population P in any year t is such that the rate of increase in the population is proportional to the population. Then ______.

a. $P = ce^{k}t$ b. $P = ce^{-kt}$ c. $P = c^{kt}$ d. P = c

19. On a multiple – choice exam with 3 possible destructives for each of the 5 equations, the probability that a student will get or more correct answers just by guessing is _______.

a. $\frac{11}{243}$ b. $\frac{3}{8}$ c. $\frac{1}{243}$ d. $\frac{5}{243}$

20. Consider a game where the player tosses a six – sided fair die. If the face that comes up is 6, the player wins ₹36, otherwise he loses ₹k^2, where k is the face that comes up k = {1, 2, 3, 4, 5}. The expected amount to win at this game in ₹ is _______.

a. $\frac{19}{6}$ b. $-\frac{19}{6}$ c. $\frac{3}{2}$ d. $-\frac{3}{2}$

PART – II

i. Answer ant SEVEN Questions. **[7 x 2 = 14]**

ii. Question Number 30 is compulsory.

21. Evaluate $\int_0^3 (3x^2 - 4x + 5)dx$

22. Solve $(1 + x^2)\frac{dy}{dx} = 1 + y^2$

23. A condition random variable x has the p.d.f defined by $f(x) = \begin{cases} Ce^{-ax}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$ the value of C if a > 0.

24. Find the value of sin-1$\left(\sin\left(\frac{5\pi}{4}\right)\right)$

25. Evaluate : $\int_0^{\pi/4} \frac{\sin^3 x}{\cos^5 x}dx$

26. Show that the following expressions is a solution of the corresponding given differential equation. $y = 2x^2$; $xy' = 2y$

27. For the distribution function given by $F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$, find the density function.

Also evaluate P(0.5 < X < 0.75).

28. Evaluate : $\int_{-\pi/4}^{\pi/4} x^3 \sin^2 x\,dx$

29. Find the constant C such that the function $f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$ is a density function.

30. If $f(x) = \begin{cases} \frac{A}{x}, & 1 < x < e^3 \\ 0, & \text{elsewhere} \end{cases}$ is a probability density function of a continuous random variable X, find $P(x > e)$.

PART - III

i. Answer any SEVEN Questions. **[7 x 3 = 21]**

ii. Question number 40 is compulsory.

31. Find the area of the region bounded by $2x - y + 1 = 0$, $y = -1$, $y = 3$ and y – axis.

32. Solve: $\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$.

33. Evaluate : $\int_0^{\frac{\pi}{2}} \sin^2 x \cos^4 x \, dx$

34. Solve the differential equation : $\frac{dy}{dx} = e^{x+y} + x^3 e^y$

35. Evaluate $\int_0^1 \frac{\sin(3 \tan^{-1} x) \tan^{-1} x}{1+x^2} dx$

36. Find the particular solution of $(1 + x^3)\, dy - x^2 y dx = 0$ satisfying the condition $y(1) = 2$.

37. If X is the random variable with distribution function F(x) given by,

$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 1, & 1 \le x \end{cases}$ then find (i) the Probability density function f(x)

38. Find the volume of the solid that results when the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b > 0)$ is revolved about the minor axis.

39. Verify that the function $y = e^{-3x}$ is a solution of the differential equation $\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 6y = 0$.

40. Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

PART – IV

Answer All the Questions: **[7 x 5 = 35]**

41. a) Solve: $(1 + 2e^{x/y})dx + 2e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$

(or)

b) Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$.

42. a) The cumulative distribution function of a discrete random variable is given by.

$$F(x) = \begin{cases} 0 & \text{for } -\infty < x < 0 \\ \frac{1}{2} & \text{for } 0 \le x < 1 \\ \frac{3}{5} & \text{for } 1 \le x < 2 \\ \frac{4}{5} & \text{for } 2 \le x < 3 \\ \frac{9}{10} & \text{for } 3 \le x < 4 \\ 1 & \text{for } 4 \le x < \infty \end{cases}$$

Find (i) the probability mass function (ii) $P(X < 3)$ and (iii) $P(X \ge 2)$.

(or)

b) Evaluate: $\int_0^{\pi/2} \frac{dx}{4+9\cos^2 x}$

43. a) Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, x – axis, $x = -3$ and $x = 3$.

(or)

b) Solve the differential equation $x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$

44. a) Evaluate: $\int_0^2 (x^2 + x + 2)dx$

(or)

b) Solve $\frac{dy}{dx} + \frac{y}{x} = \sin x$

45. a) Evaluate: $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

(or)

b) Evaluate: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$

46. a) Find the value of $\sec^2(\cot^{-1} 3) + \mathrm{cosec}^2(\tan^{-1} 2)$

(or)

b)A six sided die is marked ‘1’on the face ‘3’ on two of its faces, and ‘5’ on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find

(i) the probability mass function (ii) the cumulative distribution function

(iii) $P(4 \leq X < 10)$ (iv) $P(X \geq 6)$

47. a) Evaluate as the limits of sum $\int_1^3 (2x^2 + 5)dx$

(or)

b) Solve $x\frac{dy}{dx} + 2y - x^2 \log x = 0$

--ALL THE BEST--