



**I. Choose the correct answer**

[5 × 1 = 5]

1. If  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ , then the value of  $[\vec{a}, \vec{b}, \vec{c}]$  is  
(1)  $|\vec{a}| |\vec{b}| |\vec{c}|$  (2)  $\frac{1}{3} |\vec{a}| |\vec{b}| |\vec{c}|$  (3) 1 (4) -1
2. The volume of the parallelepiped with its edges represented by the vectors  $\hat{i} + \hat{j}$ ,  $\hat{i} + 2\hat{j}$ ,  $\hat{i} + \hat{j} + \pi\hat{k}$  is  
(1)  $\frac{\pi}{2}$  (2)  $\frac{\pi}{3}$  (3)  $\pi$  (4)  $\frac{\pi}{4}$
3. The volume of a sphere is increasing in volume at the rate of  $3\pi \text{ cm}^3 / \text{sec}$ . The rate of change of its radius when radius is  $\frac{1}{2} \text{ cm}$   
(1) 3 cm/s (2) 2 cm/s (3) 1 cm/s (4)  $\frac{1}{2} \text{ cm/s}$
4. The position of a particle moving along a horizontal line of any time  $t$  is given by  $s(t) = 3t^2 - 2t - 8$ . The time at which the particle is at rest is  
(1)  $t = 0$  (2)  $t = \frac{1}{3}$  (3)  $t = 1$  (4)  $t = 3$
5. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is  
(1) 0.2% (2) 0.4% (3) 0.04% (4) 0.08%

**II. Answer any 5 of the following questions:**

[5 × 2 = 10]

6. Prove by vector method that an angle in a semi-circle is a right angle.
7. The volume of the parallelepiped whose coterminal edges are  $7\hat{i} + \lambda\hat{j} - 3\hat{k}$ ,  $\hat{i} + 2\hat{j} - \hat{k}$ ,  $-3\hat{i} + 7\hat{j} + 5\hat{k}$  is 90 cubic units. Find the value of  $\lambda$ .
8. Determine whether the three vectors  $2\hat{i} + 3\hat{j} + \hat{k}$ ,  $\hat{i} - 2\hat{j} + 2\hat{k}$  and  $3\hat{i} + \hat{j} + 3\hat{k}$  are coplanar.
9. A particle moves along a straight line in such a way that after  $t$  seconds its distance from the origin is  $s = 2t^2 + 3t$  metres.  
(i) Find the average velocity between  $t = 3$  and  $t = 6$  seconds.  
(ii) Find the instantaneous velocities at  $t = 3$  and  $t = 6$  seconds.
10. If the mass  $m(x)$  (in kilograms) of a thin rod of length  $x$  (in metres) is given by,  $m(x) = \sqrt{3}x$  then what is the rate of change of mass with respect to the length when it is  $x = 3$  and  $x = 27$  metres.

11. Find the points on the curve  $y^2 - 4xy = x^2 + 5$  for which the tangent is horizontal.

12. Let  $f(x) = \sqrt[3]{x}$ . Find the linear approximation at  $x = 27$ . Use the linear approximation to approximate  $\sqrt[3]{27.2}$ .

**III. Answer any 7 of the following questions:**

**[7 × 5 = 35]**

13. Prove by vector method that  $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ .

14. Find the torque of the resultant of the three forces represented by  $-3\hat{i} + 6\hat{j} - 3\hat{k}$ ,  $4\hat{i} - 10\hat{j} + 12\hat{k}$  and  $4\hat{i} + 7\hat{j}$  acting at the point with position vector  $8\hat{i} - 6\hat{j} - 4\hat{k}$ , about the point with position vector  $18\hat{i} + 3\hat{j} - 9\hat{k}$ .

15. If the vectors  $a\hat{i} + a\hat{j} + c\hat{k}$ ,  $\hat{i} + \hat{k}$  and  $c\hat{i} + c\hat{j} + b\hat{k}$  are coplanar, prove that  $c$  is the geometric mean of  $a$  and  $b$ .

16. Let  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  be three non – zero vectors such that  $\vec{c}$  is a unit vectors perpendicular to both  $\vec{a}$  and  $\vec{b}$ . If the angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{6}$ , show that  $[\vec{a}, \vec{b}, \vec{c}]^2 = \frac{1}{4} |\vec{a}|^2 |\vec{b}|^2$ .

17. A particle moves along a line according to the law  $s(t) = 2t^3 - 9t^2 + 12t - 4$ , where  $t \geq 0$ .

(i) At what times the particle changes direction?

(ii) Find the total distance travelled by the particle in the first 4 seconds.

(iii) Find the particle's acceleration each time the velocity is zero.

18. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

19. Find the equation of tangent and normal to the curve given by  $x = 7 \cos t$  and  $y = 2 \sin t$ ,  $t \in \mathbb{R}$  at any point on the curve.

20. Find the angle between the rectangular hyperbola  $xy = 2$  and the parabola  $x^2 + 4y = 0$ .

21. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the following in calculating the area of the circular plate:

(i) Absolute error

(ii) Relative error

(iii) Percentage error

22. Find a linear approximation for the following functions at the indicated points.

(i)  $f(x) = x^3 - 5x + 12$ ,  $x_0 = 2$

(ii)  $h(x) = \frac{x}{x+1}$ ,  $x_0 = 1$