ST. JOSEPH STUDY CENTRE

12^{th} STD: MID TERM TEST-1 **MATHS** **Time: 2 Hrs / Total Marks: 50**

I. Answer any 5 of the following questions: **[5 x 2 = 10]**

1. Simplify the following: $\sum_{n=1}^{12} i^n$

2. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$ (i) $2z + 3w$

3. Given the complex number $z = 2 + 3i$, represent the complex number z and $z - iz$ in Argand plane.

4. If $z = -3 - 4i$, find the additive and multiplicative inverse of z.

5. Prove the Property: Re (z) = $\frac{z+\bar{z}}{2}$ and Im (z) = $\frac{z-\bar{z}}{2i}$.

6. (i) If $z = x + iy$, find Im $(3z + 4\bar{z} - 4i)$ in rectangular form

   (ii) If $|z| = 3$, show that $7 \leq |z + 6 - 8i| \leq 13$.

7. Show that the points 1, $\frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

II. Answer any 8 of the following questions: **[8 x 5 = 40]**

8. Find the values of the real numbers x and y, if the complex numbers

   $(3 - i)x - (2 - i)y + 2i + 5$ and $2x + (-1 + 2i)y + 3 + 2i$ are equal.

9. Show that

   (i) $\left(2 + i\sqrt{3}\right)^{10} + \left(2 - i\sqrt{3}\right)^{10}$ is real and

   (ii) $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary

10. Prove the property - Triangle inequality.

11.

   (i) Find the square root of $-5 - 12i$.

   (ii) Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

12. If z_1, z_2, and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that

$|9 z_1z_2 + 4 z_1z_3 + z_2z_3| = 6$.

13. Find the least value of the positive integer *n* for which $(\sqrt{3} + i)^n$ (i) real (ii) purely imaginary.

14. The complex numbers *u*, *v*, and *w* are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If v = 3 – 4i and w = 4 + 3i, find u in rectangular form.

15. Find the following

(i) $|\overline{(1 + i)}(2 + 3i)(4i - 3)|$

(ii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

16. If z_1, z_2, and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$,

find the value of $\left|\frac{1}{z1} + \frac{1}{z2} + \frac{1}{z3}\right|$.

17. If the area of the triangle formed by the vertices z , iz , and z + iz is 50 square units, find the value of $|z|$.

18. Show that the equation $z^2 = \overline{z}$ has four solutions.

---ALL THE BEST---

Test should be written under the supervision of your parents and get the answer paper signed from them.

No corrections should be made after the test timings. We expect your honesty.

Test Papers have to be submitted after the completion of all the 4 tests.

Submission Date of Test Papers: 1st August, 2nd August, 3rd August Timings: 9 AM – 12.30 PM / 5 PM- 7 PM