UNIT- 2 – BASIC ALGEBRA

I. Answer the Following (2 marks)

1. Prove that $\sqrt{3}$ is an irrational number.
2. Solve $|2x - 3| = |x - 5|$.
3. Solve $\frac{1}{|2x-1|} < 6$ and express the solution using the interval notation.
4. Solve : (i) $\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$ (ii) $\frac{5-x}{3} < \frac{x}{2} - 4$.
5. If a and b are the roots of the equation $x^2 - px + q = 0$, find the value of $\frac{1}{a} + \frac{1}{b}$.
6. Construct a quadratic equation with roots 7 and − 3.
7. If α and β are the roots of the quadratic equation $x^2 + \sqrt{2}x + 3 = 0$, form a quadratic polynomial with zeroes $\frac{1}{\alpha}, \frac{1}{\beta}$.
8. Factorize: $x^4 + 1$. (Hint: Try completing the square.)
9. Resolve the following rational expressions into partial fractions (i) $\frac{1}{x^2 - a^2}$. (ii) $\frac{1}{x^4 - 1}$.
10. Determine the region in the plane determined by the inequalities $x \leq 3y$, $x \geq y$.
11. Evaluate $(((256)^{\frac{-1}{2}})^{\frac{-1}{4}})^3$.
12. Find the logarithm of 1728 to the base $2\sqrt{3}$.
13. Compute $\log_9 27 - \log_{27} 9$.
14. Prove $\log a + \log a^2 + \log a^3 + \ldots \log a^n = \frac{n(n+1)}{2} \log a$

II. Answer the Following (3 marks)

1. A model rocket is launched from the ground. The height h reached by the rocket after t seconds from lift off is given by $h(t) = -5t^2 + 100t$, $0 \leq t \leq 20$. At what time the rocket is 495 feet above the ground?
2. A plumber can be paid according to the following schemes: In the first scheme he will be paid rupees 500 plus rupees 70 per hour, and in the second scheme he will paid rupees 120 per hour. If he works x hours, then for what value of x does the first scheme give better wages?
3. If one root of $k(x - 1)^2 = 5x - 7$ is double the other root, show that k = 2 or −25.
4. If the difference of the roots of the equation $2x^2 - (a+1)x + a - 1 = 0$ is equal to their product, then prove that a = 2.
5. Solve $2x^2 + x - 15 \leq 0$.
6. Solve $-x^2 + 3x - 2 \geq 0$.
7. Find the zeros of the polynomial function $f(x) = 4x^2 - 25$.
8. Find the real roots of $x^4 = 16$.

9. Solve $(2x + 1)^2 - (3x + 2)^2 = 0$.

10. Find all values of x for which $\frac{x^3(x-1)}{(x-2)} > 0$.

11. Resolve the following rational expressions into partial fractions (i) $\frac{(x-1)^2}{x^3+x}$ (ii) $\frac{x^2+x+1}{x^2-5x+6}$.

12. Determine the region in the plane determined by the inequalities $3x + 5y \geq 45$, $x \geq 0$, $y \geq 0$.

13. If $(x^{\frac{1}{2}} + x^{-\frac{1}{2}}) = 9/2$, then find the value of $(x^{\frac{1}{2}} - x^{-\frac{1}{2}})$ for $x > 1$.

14. Simplify and hence find the value of n: $3^{2n}9^2 3^{-n}/3^{3n} = 27$.

15. Prove $\log\frac{75}{16} - 2\log\frac{5}{9} + \log\frac{32}{243} = \log 2$.

16. If $a^2 + b^2 = 7ab$, show that $\log\frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$

17. Prove that $\log\frac{a^2}{bc} + \log\frac{b^2}{ca} + \log\frac{c^2}{ab} = 0$.

III. <u>Answer the Following(5 marks)</u>

1. Find the condition that one of the roots of $ax^2 + bx + c$ may be (i) negative of the other, (ii) thrice the other, (iii) reciprocal of the other.
2. If the equations $x^2 - ax + b = 0$ and $x^2 - ex + f = 0$ have one root in common and if the second equation has equal roots, then prove that $ae = 2(b + f)$.
3. If $x^2 + x + 1$ is a factor of the polynomial $3x^3 + 8x^2 + 8x + a$, then find the value of a.
4. Solve $\frac{x^2-4}{x^2-2x-15} \leq 0$.

5. Resolve the following rational expressions into partial fractions

   (i) $\frac{6x^2-x+1}{x^3+x^2+x+1}$ (ii) $\frac{2x^2+5x-11}{x^2+2x-3}$ (iii) $\frac{7+x}{(1+x)(1+x^2)}$

6. Determine the region in the plane determined by the inequalities $2x + y \geq 8$, $x + 2y \geq 8$, $x + y \leq 6$.
7. Solve the linear inequalities and exhibit the solution set graphically $x + y \geq 3$, $2x - y \leq 5$, $-x + 2y \leq 3$.
8. Simplify $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$.
9. If $x = \sqrt{2} + \sqrt{3}$ find $\frac{x^2+1}{x^2-2}$.
10. Given that $\log_{10} 2 = 0.30103$, $\log_{10} 3 = 0.47712$ (approximately), find the number of digits in $2^8 . 3^{12}$.
11. Prove that $\log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80} = 1$.
12. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, then prove thay xyz = 1.
13. Solve $\log_5 -x (x^2 - 6x + 65) = 2$.