ST. JOSEPH STUDY CENTRE

12th STD Quarterly Exam **MATHS** **Time: 2 Hrs 30 Mins / Total Marks: 70**

I. Answer any 10 of the following questions: [10 x 2 = 20]

1. Find the modulus and principal argument of the following complex numbers. $\sqrt{3} - i$.
2. Represent the complex number $1 + i\sqrt{3}$ in polar form.
3. Find the value of $\left(\frac{1+sin\frac{\pi}{10}+i\,cos\frac{\pi}{10}}{1+sin\frac{\pi}{10}-i\,cos\frac{\pi}{10}}\right)^{10}$
4. If $z = 2 - 2i$, find the rotation of $z = by\ \theta$ radians in the counter clockwise direction about the origin when $\theta = \frac{2\pi}{3}$.
5. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c.
6. If the equation $3x^2 + (3 - p)\,xy + qy^2 - 2px = 8\,pq$ represents a circle, find p and q. Also, determine the centre and radius of the circle.
7. Find the equation of the parabola in each of the cases given below: Vertex (1, -2) and focus (4, - 2).
8. Find the vertex, focus, equation of directrix and length of the latus rectum of the following: $y^2 = 16x$.
9. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.
10. An equation of the elliptical part of an optical lens system is $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The parabolic part of the system has a focus in common with the right focus of the ellipse .The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of the parabola.
11. Find a parametric form of vector equation of a plane which is at a distance of 7 units from the origin having 3,-4,5 as direction ratios of a normal to it.
12. Find the length of the perpendicular from the point (1, -2, 3) to plane x-y+z = 5.

II. Answer any 5 of the following questions: [5 x 3 = 15]

1. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases: $|z - 4|^2 - |z - 1|^2 = 16$.
2. Solve the equation $z^3 + 27 = 0$.
3. Find the equation of the circle through the points (1, 0), (-1, 0), and (0, 1).
4. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.
5. If a plane meets the coordinate axes at A,B,C such that the centriod of the triangle ABC is the point (u,v,w), find the equation of the plane.
6. If the straight lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{m^2}$ and $\frac{x-3}{1} = \frac{y-2}{m^2} = \frac{z-1}{2}$ are coplanar, find the distinct real values of m.

III. Answer any 7 of the following questions: **[7 x 5 = 35]**

1. If z = x + iy is a complex number such that Im $\frac{2z+1}{1z+1}$ = 0, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

2. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y^2}$, show that.

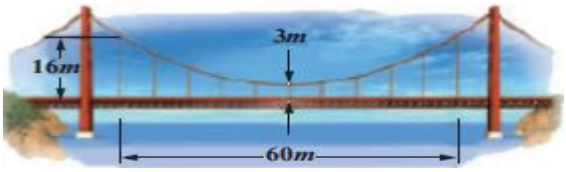
   (i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$

   (ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

   (iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

3. Two coast guard stations are located 600 km apart at points A(0, 0)and B(0, 600) . A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship

4. Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



5. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.

6. A rod of length 1 2. m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0 3. m from the end in contact with x -axis is an ellipse. Find the eccentricity
7. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(2\hat{\imath} - 7\hat{\jmath} + 4\hat{k})$=3 and 3x-5y+4z+11=0, and the point (-2, 1, 3).
8. Find the point of intersection of the line x-1= $\frac{y}{2}$ = z+1 with the plane 2x-y+2z = 2. Also, find the angle between the line and the plane.
9. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point (4, 3, 2) to the plane x+2y+3z = 2.

----------ALL THE BEST----------

70 marks will be converted to 100 marks

Test should be written under the supervision of your parents and get the answer paper signed from them.

No corrections should be made after the test timings. We expect your honesty.

Test Papers have to be submitted after the completion of all the 4 tests.

Submission Date of Test Papers: 1st October, 2nd October, 3rd October Timings: 9 AM – 12.30 PM / 5 PM- 7 PM