UNIT- 3 – TRIGONOMETRY

I. Answer the Following (2 marks)

1. If a cos θ – b sin θ = c, Show that a sin θ + b cos θ = $\pm\sqrt{a^2 + b^2 - c^2}$.
2. If y = $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$, then prove that $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ = y.
3. If $\tan^2\theta = 1 - k^2$, show that $\sec\theta + \tan^3\theta \operatorname{cosec}\theta = (2 - k^2)^{\frac{3}{2}}$.Also, Find the values of k for which this result holds.
4. Convert (i) 18^o to radians (ii) -108^o to radians.
5. Convert (i) $\frac{\pi}{5}$ radians to degrees (ii) 6 radians to degrees.
6. What must be the radius of a circular running path, around which an athlete must run 5 times in order to describe 1 km?
7. In a circle of diameter 40 cm, a chord is of length 20 cm. find the length of the minor arc of the chord.
8. Find the value of (i) $\cos 105^0$ (ii) $\sin 105^0$ (iii) $\tan\frac{7\pi}{12}$.
9. Find a quadratic equation whose roots are $\sin 15^0$ and $\cos 15^0$.
10. Show that $\tan 75^0 + \cot 75^0 = 4$.
11. Prove that
    (i) $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$
    (ii) $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$
12. Find the value of $\sin\left(22\frac{1}{2}^0\right)$.
13. Find the value of sin 2 θ, when $\sin\theta = \frac{12}{13}$, θ lies in the first quadrant.
14. Prove that $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$.
15. Prove that $1 - \frac{1}{2}\sin 2x = \frac{\sin^3 x+\cos^3 x}{\sin x+\text{cox } x}$.
16. Show that $\sin 12^0 \sin 48^0 \sin 54^0 = \frac{1}{8}$.
17. Prove that $\frac{\sin 4x + \sin 2x}{\cos 4x + \cos 2x}$ = tan 3x.
18. Find the principal solution and general solutions of the following:
    (i) $\sin\theta = -\frac{1}{\sqrt{2}}$ (ii) $\cot\theta = \sqrt{3}$ (iii) $\tan\theta = -\frac{1}{\sqrt{3}}$.
19. If the three angles in a triangle are in the ratio 1 :2:3, then prove that the corresponding sides are in the ratio 1 :$\sqrt{3}$: 2.
20. In a ΔABC, prove that (b + c) cos A + (c + a) cos B + (a + b) cos C = a + b + c.
21. In a ΔABC, prove that a cos A + b cos B + c cos C = 2a sin B sin C.
22. In a ΔABC, ∠A = 60^0. Prove that b + c = 2a $\cos\left(\frac{B-C}{2}\right)$.

23. In any ΔABC, prove that the area $\Delta = \frac{b^2 + c^2 - a^2}{4 \cot A}$.

24. Ina ΔABC, if a = 12 cm, b = 8 cm and C = 30^0, then show that its area is 24 sq.cm.

25. Ina ΔABC, if a = 18 cm, b = 24 cm and c = 30 cm, then show that its area is 216 sq.cm.

26. Find the principal value of

(i) $\operatorname{cosec}^{-1}(-1)$ (ii) $\sec^{-1}(-\sqrt{2})$ (iii) $\tan^{-1}(\sqrt{3})$.

II. Answer the Following (3 marks)

1. If sec θ + tan θ = p, obtain the values of sec θ; tan θ and sin θ in terms of p.
2. If cosec θ – sin θ = a^3 and sec θ – cos θ = b^3, then prove that $a^2b^2(a^2 + b^2) = 1$.
3. Eliminate θ from the equations a sec θ – c tan θ = b and b sec θ + d tan θ = c.
4. If in two circles, arcs of the same length subtend angles 60^0 and 75^0 at the Centre, find the ratio of their radii.
5. The perimeter of a certain sector of a circle is equal to the length of the arc of a semi-circle having the same radius. Express the angle of the sector in degrees, minutes and seconds.
6. A train is moving on a circular track of 1500 m radius at the rate of 66 km/hr. What angle will it turn in 20 seconds?
7. Find cos(x - y), given that $\cos x = -\frac{4}{5}$ with $\pi < x < \frac{3\pi}{2}$ and $\sin y = -\frac{24}{25}$ with $\pi < y < \frac{3\pi}{2}$.
8. Expand cos (A + B + C): Hence prove that cos A cos B cos C = sin A sin B cos C + sin B sin C cos A + sin C sin A cos B; if $A + B + C = \frac{\pi}{2}$.
9. Prove that cos (A + B) cos C - cos (B + C) cos A = sin B sin(C - A).
10. Prove that sin(n + 1)θ sin(n - 1) θ + cos(n + 1) θ cos(n - 1) θ = cos2θ; $n \in \mathbb{Z}$.
11. If $\tan x = \frac{n}{n+1}$ and $\tan y = \frac{1}{2n+1}$, find tan (x+ y).
12. If θ + ∅ = α and tan θ = k tan ∅, then prove that $\sin(\theta - \emptyset) = \frac{k-1}{k+1} \sin\alpha$.
13. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$, show that $\cos 3\theta = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$.
14. Prove that $\sin 4\alpha = 4 \tan\alpha \frac{1 - \tan^2\alpha}{(1 + \tan^2\alpha)^2}$.
15. Show that $\frac{\sin 8x \cos x - \sin 6x \cos 3x}{\cos 2x \cos x - \sin 3x \sin 4x} = \tan 2x$.
16. Show that $\frac{(\cos\theta - \cos 3\theta)(\sin 8\theta + \sin 2\theta)}{(\sin 5\theta - \sin\theta)(\cos 4\theta - \cos 6\theta)} = 1$.
17. Prove that sin x + sin2x + sin3x = sin2x (1 +2cos x).
18. Solve the following equations:

(i) sin 5x - sin x = cos3x (ii) $\cos\theta = \frac{\sqrt{5}+1}{4}$ (iii) $2\cos^2 x - 7\cos x + 3 = 0$.

19. In a ΔABC, prove that $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$.

20. In a ΔABC, prove that $\sin\left(\frac{B-C}{2}\right) = \frac{b-c}{a}\cos\frac{A}{2}$.

21. In a ΔABC, if $\cos C = \frac{\sin A}{2\sin B}$, show that the triangle is isosceles.

22. In a ΔABC, prove that $\frac{\sin B}{\sin C} = \frac{c - a\cos B}{b - a\cos C}$.

III. Answer the Following(5 marks)

1. If $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\theta$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{2n}\theta$, $0 < \theta < \frac{\pi}{2}$, then show that $xyz = x + y + z$.

   [Hint: Use the formula $1 + x + X^2 + X^3 + \ldots\ldots = \frac{1}{1-x}$, where $|x| < 1$.]

2. If $\frac{\cos^4\alpha}{\cos^2\beta} + \frac{\sin^4\alpha}{\sin^2\beta} = 1$, prove that (i) $\sin^4\alpha + \sin^4\beta = 2\sin^2\alpha\sin^2\beta$ (ii) $\frac{\cos^4\beta}{\cos^2\alpha} + \frac{\sin^4\beta}{\sin^2\alpha} = 1$.

3. If $\cot\theta(1 + \sin\theta) = 4m$ and $\cot\theta(1 - \sin\theta) = 4n$, then prove that $(m^2 - n^2)^2 = mn$.

4. Prove that $\frac{\cot(180^0+\theta)\sin(90^0-\theta)\cos(-\theta)}{\sin(270^0+\theta)\tan(-\theta)\operatorname{cosec}(360^0+\theta)} = \cos^2\theta\cot\theta$.

5. Show that $\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2$.

6. Show that $\cos^2 A + \cos^2 B - 2\cos A\cos B\cos(A + B) = \sin^2(A + B)$.

7. If $\cos(\alpha-\beta) + \cos(\beta-\gamma) + \cos(\gamma-\alpha) = -\frac{3}{2}$ then prove that $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$.

8. If $A + B = 45^0$, show that $(1 + \tan A)(1 + \tan B) = 2$.

9. Prove that $\tan\left(\frac{\pi}{4} + \theta\right) - \tan\left(\frac{\pi}{4} - \theta\right) = 2\tan 2\theta$.

10. Prove that $\frac{\sin x+\sin 3x+\sin 5x+\sin 7x}{\cos x+\cos 3x+\cos 5x+\cos 7x} = \tan 4x$.

11. Prove that $\frac{\sin(4A-2B) + \sin(4B-2A)}{\cos(4A-2B) + \cos(4B-2A)} = \tan(A + B)$.

12. Show that $\cot(A + 15^0) - \tan(A - 15^0) = \frac{4\cos 2A}{1+2\sin 2A}$.

13. If $A + B + C = 180^0$, prove that

    (i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

    (ii) $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$.

    (iii) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A\cos B\cos C$

    (iv) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$.

    (v) $\text{Sin } A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

    (vi) $\sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C) = 4\sin A\sin B\sin C$.

14. If x + y + z = xyz, then prove that $\frac{2x}{1-x^2} + \frac{2y}{1-y^2} + \frac{2x}{1-z^2} = \frac{2x}{1-x^2}\frac{2y}{1-y^2}\frac{2x}{1-z^2}$.

15. If A + B + C = $\frac{\pi}{2}$, prove the following

    (i) sin 2A + sin2B + sin2C = 4cos A cos B cos C

    (ii) cos 2A + cos2B + cos2C = 1+4sin A sin B sin C.

16. In a ΔABC, if $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-c)}$, prove that a^2, b^2, c^2 are in Arithmetic Progression.

17. In a ΔABC, prove the following

    (i) $\frac{a\sin(B-C)}{b^2-c^2} = \frac{b\sin(C-A)}{c^2-a^2} = \frac{c\sin(A-B)}{a^2-b^2}$

    (ii) $\frac{a+b}{a-b} = \tan\left(\frac{A+B}{2}\right)\cot\left(\frac{A+B}{2}\right)$

18. A farmer wants to purchase a triangular shaped land with sides 120feet and 60feet and the angle included between these two sides is 60^0. If the land costs ₹500 per sq.ft ,find the amount he needed to purchase the land . Also find the perimeter of the land.

19. A fighter jet has to hit a small target by flying a horizontal distance. When the target is sighted, the pilot measures the angle of depression to be 30^0. If after 100 km, the target has an angle of depression of 60^0, how far is the target from the fighter jet at that instant?

20. A plane is 1 km from one land mark and 2 km from another. From the planes point of view the land between them subtends an angle of 45^0. How far apart are the landmarks?