UNIT- 5 – BINOMIAL THEROEM (Ex 5.1 – Ex 5.2)

I. Answer the Following (2 marks)

1. Find the expansion of $(2x + 3)^5$.
2. Evaluate 98^4.
3. Find the middle term in the expansion of $(x + y)^6$.
4. Find the last two digits of the number 7^{400}.
5. Find the last two digits of the number 3^{600}.

II. Answer the Following (3 marks)

1. Find the middle terms in the expansion of $(x + y)^7$.
2. Find the coefficient of x^6 in the expansion of $(3 + 2x)^{10}$.
3. Expand $\left(x^2 + \sqrt{1 - x^2}\right)^5 + \left(x^2 + \sqrt{1 - x^2}\right)^5$.
4. Using binomial theorem, indicate which of the following two number is larger: $(1:01)^{1000000}$, 10000.
5. Find the coefficient of x^{15} in $\left(x^2 + \frac{1}{x^3}\right)^{10}$.
6. Find the coefficient of x^2 and the coefficient of x^6 in $\left(x^2 - \frac{1}{x^3}\right)^6$.
7. Find the constant term of $\left(2x^3 - \frac{1}{3x^2}\right)^5$.
8. Prove that $C_0^2 + C_1^2 + C_2^2 + \cdots C_n^2 = \frac{(2n)!}{(n!)^2}$.
9. Write the n^{th} term of the following sequences.

   (i) 2, 2, 4, 4, 6, 6, ……… (ii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots\ldots$ (iii) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}, \ldots$

   (iv) 6, 10, 4, 12, 2, 14, 0, 16, -2, …….
10. Write the n^{th} term of the sequence $\frac{2}{1^2 2^2}, \frac{5}{2^2 3^2}, \frac{7}{3^2 4^2}$, ……...as a difference of two terms.
11. If t_k is the kth term of a GP, then show that t_{n-k}, t_n, t_{n+k} also form a GP for any positive integer k.

Answer the Following (5 marks)

1. The 2^{nd}, 3^{rd} and 4^{th} terms in the binomial expansion of $(x + a)^n$ are 240, 720 and 1080 for a suitable value of x. Find x, a and n.
2. Using Binomial theorem, prove that $6^n - 5n$ always leaves remainder 1 when divided by 25 for all positive integer n.
3. Find the coefficient of x^4 in the expansion of $(1 + x^3)^{50}\left(x^2 + \frac{1}{x}\right)^5$.

4. If a and b are distinct integers, prove that a - b is a factor of $a^n - b^n$, whenever n is a positive integer.[Hint :write $a^n = (a - b + b)^n$ and expand]
5. In the binomial expansion of $(a + b)^n$, if the coefficients of the 4th and 13th terms are equal then, find n.
6. If the binomial coefficients of three consecutive terms in the expansion of $(a + x)^n$ are in the ratio 1 : 7 : 42; then find n.
7. In the binomial expansion of $(1+x)^n$, the coefficients of the 5th, 6th and 7th terms are in AP .Find all values of n.
8. Write the first 6 terms of the sequences whose nth term a_n is given below.

(i) $$a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$$

(ii) $$a_n = \begin{cases} 1 & \text{if } n = 1 \\ 2 & \text{if } n = 2 \\ a_{n-1} + a_{n-2} & \text{if } n > 2 \end{cases}$$

(iii) $$a_n = \begin{cases} n & \text{if } n \text{ is } 1, 2 \text{ or } 3 \\ a_{n-1} + a_{n-2} + a_{n-2} & \text{if } n > 3 \end{cases}$$

9. The AM of two numbers exceeds their GM by 10 and HM by 16. Find the numbers.
10. If the roots of the equation $(q - r)x^2 + (r - p)x + p - q = 0$ are equal, then show that p; q and r are in AP.
11. If a, b, c are respectively the pth, qth and rth terms of a GP, show that (q - r) log a + (r - p) log b + (p - q) log c = 0: