



**I. Answer any 5 of the following questions:**

[5 x 2 = 10]

- Suppose two coins are tossed once. If X denotes the number of tails, (i) write down the sample space (ii) find the inverse image of 1 (iii) the values of the random variable and number of elements in its inverse images.
- Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.
- Compute  $P(X = k)$  for the binomial distribution,  $B(n, p)$  where  $n = 6, p = \frac{1}{3}, k = 3$ .
- Fill in the following table so that the binary operation  $*$  on  $A = \{a, b, c\}$  is commutative.

*	a	b	c
a	b		
b	c	b	a
c	a		c

- Let  $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ , be any three boolean matrices of the same type.

Find (i)  $A \vee B$       (ii)  $A \wedge B$       (iii)  $(A \vee B) \wedge C$       (iv)  $(A \wedge B) \vee C$ .

- Show that  $p \rightarrow q$  and  $q \rightarrow p$  are not equivalent.

**II. Answer any 5 of the following questions:**

[5 x 3 = 15]

- A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find

- the probability mass function
- the cumulative distribution function
- $P(4 \leq X < 10)$
- $P(X \geq 6)$

- The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function

$$F(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the expected life of this electronic equipment.

- Using binomial distribution find the mean and variance of X for the following experiments

- A fair coin is tossed 100 times, and X denote the number of heads.

- On  $\mathbb{Z}$ , define  $*$  by  $(m * n) = m^n + n^m : \forall m, n \in \mathbb{Z}$ . Is  $*$  binary on  $\mathbb{Z}$ ?

- Establish the equivalence property connecting the bi-conditional with conditional:  $p \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

12. Determine the truth value of each of the following statements

- (i) If  $6 + 2 = 5$ , then the milk is white.
- (ii) China is in Europe or  $\sqrt{3}$  is an integer
- (iii) It is not true that  $5 + 5 = 9$  or Earth is a planet
- (iv) 11 is a prime number and all the sides of a rectangle are equal

**III. Answer any 5 of the following questions:**

[5 x 5 = 25]

13. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If  $X$  denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.

14. A random variable  $X$  has the following probability mass function.

$x$	1	2	3	4	5
$f(x)$	$k^2$	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of  $k$  (ii)  $P(2 \leq X < 5)$  (iii)  $P(3 < X)$

15. Find the constant  $C$  such that the function  $f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{otherwise} \end{cases}$  is a density function, and compute (i)  $P(1.5 < X < 3.5)$  (ii)  $P(X \leq 2)$  (iii)  $P(3 < X)$ .

16. Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation  $\times_{11}$  on a subset  $A = \{1,3,4,5,9\}$  of the set of remainders  $\{0,1,2,3,4,5,6,7,8,9,10\}$ .

17. i) Let  $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\} \right\}$  and let  $*$  be the matrix multiplication. Determine whether  $M$  is closed under  $*$ . If so, examine the commutative and associative properties satisfied by  $*$  on  $M$ .

(ii) Let  $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\} \right\}$  and let  $*$  be the matrix multiplication. Determine whether  $M$  is closed under  $*$ . If so, examine the existence of identity, existence of inverse properties for the operation  $*$  on  $M$

18.

- i. Let  $A$  be  $\mathbb{Q} \setminus \{1\}$ . Define  $*$  on  $A$  by  $x * y = x + y - xy$ . Is  $*$  binary on  $A$ ? If so, examine the commutative and associative properties satisfied by  $*$  on  $A$ .
- ii. Let  $A$  be  $\mathbb{Q} \setminus \{1\}$ . Define  $*$  on  $A$  by  $x * y = x + y - xy$ . Is  $*$  binary on  $A$ ? If so, examine the existence of identity, existence of inverse properties for the operation  $*$  on  $A$ .

-----ALL THE BEST-----

Test should be written under the supervision of your parents and get the answer paper signed from them.

No corrections should be made after the test timings. We expect your honesty.

Test Papers have to be submitted after the completion of all the 4 tests.

Submission Date of Test Papers: 1<sup>st</sup> December, 2<sup>nd</sup> December, 3<sup>rd</sup> December Timings: 9 AM – 12.30 PM / 5 PM- 7 PM