## CHAPTER - 2 - NUMBERS AND SEQUENCES

### I. Answer the Following (2 marks)

1. Show that the square of an odd integer is of the form 4q +1, for some integer q.

2. If the Highest Common Factor of 210 and 55 is expressible in the form 55x -325, find x.

3. Find all positive integers, when divided by 3 leaves remainder 2.

4. Find the remainders when 70004 and 778 is divided by 7.

5. Determine the value of d such that $15 \equiv 3 \pmod{d}$.

6. Solve $8x \equiv 1 \pmod{11}$

7. Compute x, such that $10^4 \equiv x \pmod{19}$

8. What is the time 100 hours after 7 a.m.?

9. What is the time 15 hours before 11 p.m.?

10. The general term of a sequence is defined as $a_n = \begin{cases} n(n+3) ; & n \in \text{ is odd} \\ n^2 + 1 ; & n \in \text{ is even} \end{cases}$

    Find the eleventh and eighteenth terms.

11. Find the first five terms of the following sequence. $a_1 = 1, a_2 = 1, a_n = \dfrac{a_{n-1}}{a_{n-2}+3} ; n \geq 3, n \in \mathbb{N}$.

12. Find $a_s$ and $a_{15}$ whose $n^{th}$ term is $a_n = \begin{cases} \dfrac{n^2-1}{n+3} ; & n \text{ is even}, n \in \mathbb{N} \\ \dfrac{n^2}{2n+1} ; & n \text{ is odd}, n \in \mathbb{N} \end{cases}$

13. If $a_1 = 1, a_2 = 1,$ and $a_n = 2a_{n-1} + a_{n-2}$ $n \geq 3, n \in \mathbb{N}$, then find the first six terms of the sequence.

14. Find the $n^{th}$ term of the following sequences (i) 2, 5, 10, 17, ........  (ii)  3, 8, 13, 18, ........

15. First term a and common difference d are given below. Find the corresponding A.P.

    (i)  a = 5 , d = 6          (ii) a = 7 , d =− 5

16. Find the $19^{th}$ term of an A.P. -11,-15,-19,...

17. Which term of an A.P. 16, 11, 6, 1,... is -54 ?

18. Find the middle term(s) of an A.P. 9, 15, 21, 27,…,183.

19. How many terms of the series 1+ 5 + 9 +... must be taken so that their sum is 190?

20. Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

21. Find the number of terms in the following G.P.

    (i)   4, 8, 16, …..8192?          (ii) $\dfrac{1}{3}, \dfrac{1}{9}, \dfrac{1}{27}, \ldots \ldots \dfrac{1}{2187}$

22. Find the sum $3 + 1 + \dfrac{1}{3} + \cdots \infty$.

23. If the first term of an infinite G.P. is 8 and its sum to infinity is $\dfrac{32}{3}$ then find the common ratio.

24. Find the rational form of the number $0.\overline{123}$.

25. Find the sum of the following series

(i)   $1+2+3+\ldots+60$          (ii)   $6^2+7^2+8^2+\ldots+21^2$          (iii) $10^3+11^3+12^3+\ldots+20^3$

## II.  Answer the Following (3 marks)

1. Prove that the product of two consecutive positive integers is divisible by 2.

2. If $13824 = 2^a \times 3^b$ then find a and b.

3. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where $p_1, p_2, p_3, p_4$ are primes in ascending order and $x_1, x_2, x_3, x_4$ are integers,

   find the value of $p_1, p_2, p_3, p_4$ and $x_1, x_2, x_3, x_4$.

4. Find the least number that is divisible by the first ten natural numbers.

5. Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming?

6. Prove that $2^n + 6 \times 9^n$ is always divisible by 7 for any positive integer n.

7. Find the remainder when $2^{81}$ is divided by 17.

8. Find the $15^{th}$, $24^{th}$ and $n^{th}$ term (general term) of an A.P. given by 3, 15,27, 39,…

9. If 3 +k, 18 -k, 5k +1 are in A.P. then find k.

10. Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

11. How many consecutive odd integers beginning with 5 will sum to 480?

12. Find the sum of first 28 terms of an A.P. whose $n^{th}$ term is 4n -3.

13. In a G.P. 729, 243, 81,… find $t_7$.

14. Find x so that x + 6, x + 12 and x + 15 are consecutive terms of a Geometric Progression.

15. If $1+2+3+\ldots+k = 325$, then find $1^3+2^3+3^3+\ldots+k^3$.

16. If $1^3+2^3+3^3+\ldots+k^3 = 44100$ then find $1+2+3+\ldots+k$.

## III.  Answer the Following (5 marks)

1. When the positive integers a, b and c are divided by 13, the respective remainders are 9,7 and 10. Show that a+b+c is

   divisible by 13.

2. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of

   (i)   340 and 412          (ii) 867 and 255          (iii) 10224 and 9648          (iv) 84, 90 and 120

3. If d is the Highest Common Factor of 32 and 60, find x and y satisfying $d = 32x + 60y$.

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4. A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11?

5. Find the greatest number consisting of 6 digits which is exactly divisible by 24, 15, 36?

6. Find the least positive value of x such that

   (i) $67 + x \equiv 1 \pmod 4$                        (ii) $98 \equiv (x + 4) \pmod 5$

7. The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23:30 hours. If the time at Chennai is four and half hours ahead to that of London's time, then find the time at London, when will the flight lands at London Airport.

8. If $l^{th}$, $m^{th}$ and $n^{th}$ terms of an A.P. are x, y, z respectively, then show that

   (i) $x(m-n) + y(n-l) + z(l-m) = 0$          (ii) $(x-y)n + (y-z)l + (z-x)m = 0$

9. In an A.P., sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers.

10. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

11. The ratio of $6^{th}$ and $8^{th}$ term of an A.P. is 7:9. Find the ratio of $9^{th}$ term to $13^{th}$ term.

12. Priya earned ₹15,000 in the first month. Thereafter her salary increased by ₹1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹900 per year. How long will it take for her to save ₹20,000 per month.

13. A mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. The tiles are alternate in colour as shown in the figure. Find the number of tiles of each colour and total number of tiles in the mosaic.
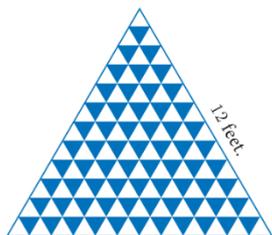


14. The sum of first n terms of a certain series is given as $2n^2 - 3n$. Show that the series is an A.P.

15. The $104^{th}$ term and $4^{th}$ term of an A.P. are 125 and 0. Find the sum of first 35 terms.

16. Find the sum of all odd positive integers less than 450.

17. A man repays a loan of ₹65,000 by paying ₹400 in the first month and then increasing the payment by ₹300 every month. How long will it take for him to clear the loan?

18. A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step.

   i. How many bricks are required for the top most step?

   ii. How many bricks are required to build the stair case?

19. Find the sum $\left[\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots \cdots \text{to 12 terms}\right]$.

20. In a G.P. the $9^{th}$ term is 32805 and $6^{th}$ term is 1215. Find the $12^{th}$ term.

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21. Find the $10^{th}$ term of a G.P. whose $8^{th}$ term is 768 and the common ratio is 2.

22. Sivamani is attending an interview for a job and the company gave two offers to him.

    Offer A: ₹20,000 to start with followed by a guaranteed annual increase of 6% for the first 5 years.

    Offer B: ₹22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years.

    What is his salary in the 4th year with respect to the offers A and B?

23. If a, b, c are three consecutive terms of an A.P. and x, y, z are three consecutive terms of a G.P. then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$ .

24. Find the sum to n terms of the series

    (i) $0.4 + 0.44 + 0.444 + \ldots$ to n terms       (ii) $3 + 33 + 333 + \ldots$ to n terms

25. Find the sum of the Geometric series $3 + 6 + 12 + \ldots + 1536$.

26. How many terms of the series $1^3 + 2^3 + 3^3 + \ldots$ should be taken to get the sum 14400?

27. The sum of the cubes of the first n natural numbers is 2025, then find the value of n.

28. Find the sum of the series $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 15^3) + \ldots$ to

    (i) n terms                                          (ii) 8 terms