CHAPTER - 12 – INTRODUCTION TO PROBABILITY THEORY

I. Answer the Following (2 marks)

1. (i) The odds that the event A occurs is 5 to 7, find P (A).

   (ii) Suppose P (B) = $\frac{2}{5}$. Express the odds that the event B occurs.

2. If A and B are two events such that P (A∪ B) = 0.7, P (A ∩ B) = 0.2, and P (B) = 0.5, then show that A and B are independent.

3. If A and B are two independent events such that P (A∪B) = 0.6, P (A) = 0.2, find P (B).

4. If P (A) =0.5, P (B) =0.8 and P (B/A) = 0.8, find P (A / B) and P (A∪B .

II. Answer the Following (3 marks)

1. If two coins are tossed simultaneously, then find the probability of getting

   (i) one head and one tail (ii) at most two tails

2. What is the chance that

   (i) non-leap year (ii) leap year should have fifty three Sundays?

3. An integer is chosen at random from the first 100 positive integers. What is the probability that the integer chosen is a prime or multiple of 8?

4. If A and B are two events associated with a random experiment for which P(A) = 0.35, P(A or B) = 0.85, and P(A and B) = 0.15. Find (i) P(only B) (ii) P($\bar{B}$) (iii) P(only A)

5. A die is thrown twice. Let A be the event, 'First die shows 5' and B be the event, 'second die shows 5'. Find P(A∪B) .

6. If for two events A and B, P (A) = $\frac{3}{4}$, P(B) = $\frac{2}{5}$ and A ∪ B= S (sample space),

   find the conditional probability P(A / B).

7. The probability that a car being filled with petrol will also need an oil change is 0.30; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.15.

   (i) If the oil had to be changed, what is the probability that a new oil filter is needed?

   (ii) If a new oil filter is needed, what is the probability that the oil has to be changed?

8. One bag contains 5 white and 3 black balls. Another bag contains 4 white and 6 black balls. If one ball is drawn from each bag, find the probability that (i) both are white (ii) both are black (iii) one white and one black.

III. Answer the Following (5 marks)

1. Eight coins are tossed once, find the probability of getting

   (i) exactly two tails (ii) atleast two tails (iii) atmost two tails

2. A bag contains 7 red and 4 black balls, 3 balls are drawn at random. Find the probability that (i) all are red (ii) one red and 2 black.

3. A cricket club has 16 members, of whom only 5 can bowl. What is the probability that in a team of 11 members at least 3 bowlers are selected?

4. Given that P(A) = 0.52, P(B) = 0.43, and P(A ∩ B) = 0.24, find

   (i) $P(A \cap \bar{B})$ (ii) $P(A \cup B)$ (iii) $P(\bar{A} \cap \bar{B})$ (iv) $P(\bar{A} \cup \bar{B})$.

5. A town has 2 fire engines operating independently. The probability that a fire engine is available when needed is 0.96.

   (i) What is the probability that a fire engine is available when needed?

   (ii) What is the probability that neither is available when needed?

6. The probability that a new railway bridge will get an award for its design is 0.48, the probability that it will get an award for the efficient use of materials is 0.36, and that it will get both awards is 0.2. What is the probability, that

   (i) it will get at least one of the two awards (ii) it will get only one of the awards.

7. Two cards are drawn from a pack of 52 cards in succession. Find the probability that both are Jack when the first drawn card is (i) replaced (ii) not replaced

8. X speaks truth in 70 percent of cases, and Y in 90 percent of cases. What is the probability that they likely to contradict each other in stating the same fact?

9. Given P (A) = 0.4 and P (A ∪ B) = 0.7. Find P(B) if

   (i) A and B are mutually exclusive (ii) A and B are independent events

   (iii) P (A / B) = 0.4 (iv) P(B / A) = 0.5

10. A year is selected at random. What is the probability that

    (i) it contains 53 Sundays (ii) it is a leap year which contains 53 Sundays

11. Suppose the chances of hitting a target by a person X is 3 times in 4 shots, by Y is 4 times in 5 shots, and by Z is 2 times in 3 shots. They fire simultaneously exactly one time. What is the probability that the target is damaged by exactly 2 hits?

12. A factory has two Machines-I and II. Machine-I produces 60% of items and Machine-II produces 40% of the items of the total output. Further 2% of the items produced by Machine-I are defective whereas 4% produced by Machine-II are defective. If an item is drawn at random what is the probability that it is defective?

13. There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?

14. A firm manufactures PVC pipes in three plants viz, X, Y and Z. The daily production volumes from the three firms X, Y and Z are respectively 2000 units, 3000 units and 5000 units. It is known from the past experience that 3% of the output from plant X, 4% from plant Y and 2% from plant Z are defective. A pipe is selected at random from a day's total production,

    (i) find the probability that the selected pipe is a defective one.

    (ii) if the selected pipe is a defective, then what is the probability that it was produced by plant Y ?

15. The chances of A, B and C becoming manager of a certain company are 5 : 3 : 2. The probabilities that the office canteen will be improved if A, B, and C become managers are 0.4, 0.5 and 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager?

16. An advertising executive is studying television-viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television; the wife is also watching the television.