

CHAPTER - 7 – MATRICES AND DETERMINANTS
I. Answer the Following (2 marks)

1. Find x , y , a , and b if $\begin{bmatrix} 3x + 4y & 6 & x - 2y \\ a + b & 2a - b & -3 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 4 \\ 5 & -5 & -3 \end{bmatrix}$.

2. Compute $A + B$ and $A - B$ if $A = \begin{bmatrix} 4 & \sqrt{5} & 7 \\ -1 & 0 & 0.5 \end{bmatrix}$ and $B = \begin{bmatrix} \sqrt{3} & \sqrt{5} & 7.3 \\ 1 & \frac{1}{3} & \frac{1}{4} \end{bmatrix}$.

3. Simplify: $\sec \theta \begin{bmatrix} \sec \theta & \tan \theta \\ \tan \theta & \sec \theta \end{bmatrix} - \tan \theta \begin{bmatrix} \tan \theta & \sec \theta \\ \sec \theta & \tan \theta \end{bmatrix}$.

4. Solve for x if $\begin{bmatrix} x & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ -1 & -4 & 1 \\ -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ 1 \end{bmatrix} = 0$.

5. Find the values of p , q , r , and s if $\begin{bmatrix} p^2 - 1 & 0 & -31 - q^3 \\ 7 & r + 1 & 9 \\ -2 & 8 & s - 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -4 \\ 7 & \frac{3}{2} & 9 \\ -2 & 8 & -\pi \end{bmatrix}$.

6. Determine the value of $x + y$ if $\begin{bmatrix} 2x + y & 4x \\ 5x - 7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y - 13 \\ y & x + 6 \end{bmatrix}$.

7. For what value of x , the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & x^3 \\ 2 & -3 & 0 \end{bmatrix}$ is skew-symmetric.

8. Find $|A|$ if $A = \begin{bmatrix} 0 & \sin \alpha & \cos \alpha \\ \sin \alpha & 0 & \sin \beta \\ \cos \alpha & -\sin \beta & 0 \end{bmatrix}$.

9. Without expanding the determinant, prove that $\begin{vmatrix} s & a^2 & b^2 + c^2 \\ s & b^2 & c^2 + a^2 \\ s & c^2 & a^2 + b^2 \end{vmatrix} = 0$.

10. Show that $\begin{vmatrix} b + c & bc & b^2c^2 \\ c + a & ca & c^2a^2 \\ a + b & ab & a^2b^2 \end{vmatrix} = 0$.

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11. If $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$, prove that $\sum_{k=1}^n \det(A^k) = \frac{1}{3} \left(1 - \frac{1}{4^n} \right)$.

12. Without expanding, evaluate the following determinants:

(i) $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

(ii) $\begin{vmatrix} x + y & y + z & z + x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

13. Show that $\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix}$.

 14. Find the area of the triangle whose vertices are $(-2, -3)$, $(3, 2)$, and $(-1, -8)$.

15. Determine the values of a and b so that the following matrices are singular: $A = \begin{vmatrix} 7 & 3 \\ -2 & a \end{vmatrix}$.

II. Answer the Following (3 marks)

1. Construct an $m \times n$ matrix is given by $A = [a_{ij}]$, where a_{ij} is given by

(i) $a_{ij} = \frac{(i-2j)^2}{2}$ with $m = 2, n = 3$ (ii) $a_{ij} = \frac{|3i-4j|}{4}$ with $m = 3, n = 4$

2. Consider the matrix $A_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$.

(i) Show that $A_\alpha A_\beta = A_{(\alpha + \beta)}$

(ii) Find all possible real values of α satisfying the condition $A_\alpha + A_\alpha^T = I$.

3. If A is a square matrix such that $A^2 = A$, find the value of $7A - (I + A)^3$.

4. Find the matrix A which satisfies the matrix relation $A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

5. Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A^T = \begin{bmatrix} -1 & -8 & -10 \\ 1 & 2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$.

6. A shopkeeper in a Nuts and Spices shop makes gift packs of cashew nuts, raisins and almonds.

Pack I contains 100 gm of cashew nuts, 100 gm of raisins and 50 gm of almonds.

Pack-II contains 200 gm of cashew nuts, 100 gm of raisins and 100 gm of almonds.

Pack-III contains 250 gm of cashew nuts, 250 gm of raisins and 150 gm of almonds.

The cost of 50 gm of cashew nuts is ₹ 50, 50 gm of raisins is ₹10, and 50 gm of almonds is ₹ 60. What is the cost of each gift pack?

7. Compute $|A|$ using Sarrus rule if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$.

8. If a, b, c and x are positive real numbers, then show that $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix}$ is zero.

9. If a, b, c are all positive, and are $p^{\text{th}}, q^{\text{th}}$ and r^{th} terms of a G.P., show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$.

10. Find the value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ if $x, y, z \neq 1$.

11. Solve the following problems by using Factor Theorem : show that $\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix} = (x - a)^2 (x + 2a)$.

12. Solve the following problems by using Factor Theorem : Show that $\begin{vmatrix} b + c & a - c & a - b \\ b - c & c + a & b - a \\ c - b & c - a & a + b \end{vmatrix} = 8abc$.

13. Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1 - 2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^{-2} - 2x \\ -x^2 & x^2 - 2x & -1 \end{vmatrix}$.

14. Show that the points $(a, b + c)$, $(b, c + a)$, and $(c, a + b)$ are collinear.
15. Find the area of the triangle whose vertices are $(0, 0)$, $(1, 2)$ and $(4, 3)$.
16. If $(k, 2)$, $(2, 4)$ and $(3, 2)$ are vertices of the triangle of area 4 square units then determine the value of k .
17. Determine the values of a and b so that the following matrices are singular: $B = \begin{bmatrix} b-1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & -2 & 4 \end{bmatrix}$.
18. If $\cos 2\theta = 0$, determine $\begin{vmatrix} 0 & \cos \theta & \sin \theta \\ \cos \theta & \sin \theta & 0 \\ \sin \theta & 0 & \cos \theta \end{vmatrix}^2$
19. Find the value of the product ; $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$.

III. Answer the Following (5 marks)

1. Without expanding the determinants, show that $|B| = 2|A|$. When $B = \begin{bmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{bmatrix}$ and $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$.
2. Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$.
3. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$.
4. If $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$. Prove that a, b, c are in G.P. or α is a root of $ax^2 + 2bx + c = 0$.
5. Prove that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$.
6. Show that $\begin{vmatrix} a^2 + x^2 & ab & ac \\ ab & b^2 + x^2 & bc \\ ac & bc & c^2 + x^2 \end{vmatrix}$ is divisible by x^4 .
7. If $\lambda = -2$ determine the value of $\begin{vmatrix} 0 & 2\lambda & 1 \\ \lambda^2 & 0 & 3\lambda^2 + 1 \\ -1 & 6\lambda - 1 & 0 \end{vmatrix}$.
8. Determine the roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$.
9. Verify that $\det(AB) = (\det A) (\det B)$ for $A = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix}$.
10. Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x - y)(y - z)(z - x)(xy + yz + zx)$.

11. Prove that $|A| = \begin{vmatrix} (q+r)^2 & p^2 & p^2 \\ q^2 & (r+p)^2 & q^2 \\ r^2 & r^2 & (p+q)^2 \end{vmatrix} = 2pqr(p+q+r)^3$.

12. Solve the following problems by using Factor Theorem :

Show that $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$.

13. Solve the following problems by using Factor Theorem : Solve $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$.

14. Solve the following problems by using Factor Theorem : Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$.

15. If A_1, B_1, C_1 are the cofactors of a_1, b_1, c_1 , respectively, $i = 1$ to 3 in

$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, show that $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = |A|^2$.

