CHAPTER - 8 – VECTOR ALGEBRA

I. Answer the Following (2 marks)

1. Let $\vec{a}$ and $\vec{b}$ be the position vectors of the points A and B. Prove that the position vectors of the points which trisects the line segment AB are $\frac{\vec{a}+2\vec{b}}{3}$ and $\frac{\vec{b}+2\vec{a}}{3}$.
2. If D and E are the midpoints of the sides AB and AC of a triangle ABC, prove that $\overrightarrow{BE}+\overrightarrow{DC}=\frac{3}{2}\overrightarrow{DC}$.
3. If $\vec{a}$ and $\vec{b}$ represent a side and a diagonal of a parallelogram, find the other sides and the other diagonal.
4. If $\overrightarrow{PO}+\overrightarrow{OQ}=\overrightarrow{QO}+\overrightarrow{OR}$, prove that the points P, Q, R are collinear.
5. A triangle is formed by joining the points (1, 0, 0), (0, 1, 0) and (0, 0, 1). Find the direction cosines of the medians.
6. If (a , a + b , a + b + c) is one set of direction ratios of the line joining (1, 0, 0) and (0, 1, 0), then find a set of values of a, b, c.
7. Show that the vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}$, $3\hat{\imath}-4\hat{\jmath}-4\hat{k}$, $\hat{\imath}-3\hat{\jmath}-5\hat{k}$ form a right angled triangle.
8. Find the value of λ for which the vectors $\hat{a}=3\hat{\imath}+2\hat{\jmath}+9\hat{k}$ and $\hat{b}=\hat{\imath}+\lambda\hat{\jmath}+3\hat{k}$ are parallel.
9. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ prove that $\vec{a}$ and $\vec{b}$ are perpendicular.
10. For any vector $\vec{r}$ prove that $\vec{r}=(\vec{r}\cdot\hat{\imath})\,\hat{\imath}+(\vec{r}\cdot\hat{\jmath})\,\hat{\jmath}+(\vec{r}\cdot\hat{k})\,\hat{k}$
11. If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are three unit vectors satisfying $\vec{a}-\sqrt{3}\,\vec{b}+\vec{c}=\vec{0}$ then find the angle between $\vec{a}$ and $\vec{c}$.
12. Find the value λ for which the vectors $\vec{a}$ and $\vec{b}$ are perpendicular, where
    (i) $\vec{a}=2\hat{\imath}+\lambda\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}-2\hat{\jmath}+3\hat{k}$ (ii) $\vec{a}=2\hat{\imath}+4\hat{\jmath}-\hat{k}$ and $\vec{b}=3\hat{\imath}-2\hat{\jmath}+\lambda\hat{k}$.
13. Find the angle between the vectors (i) $2\hat{\imath}+3\hat{\jmath}-6\hat{k}$ and $6\hat{\imath}-3\hat{\jmath}+2\hat{k}$ (ii) $\hat{\imath}-\hat{\jmath}$ and $\hat{\jmath}-\hat{k}$
14. Find the area of the parallelogram whose adjacent sides are $\vec{a}=3\hat{\imath}+\hat{\jmath}+4\hat{k}$ and $\vec{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$.
15. Find the area of a triangle having the points A(1,0,0), B(0,1,0), and C(0,0,1) as its vertices.

II. Answer the Following (3 marks)

1. If $\vec{a}$ and $\vec{b}$ are vectors represented by two adjacent sides of a regular hexagon, then find the vectors represented by other sides.
2. Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side whose length is half of the length of the third side.
3. If D is the midpoint of the side BC of a triangle ABC, prove that $\overrightarrow{AB}-\overrightarrow{AC}=2\overrightarrow{AD}$.
4. If G is the centroid of a triangle ABC, prove that $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\vec{0}$.
5. Prove that the points whose position vectors $2\hat{\imath}+4\hat{\jmath}+3\hat{k}$, $4\hat{\imath}+\hat{\jmath}+9\hat{k}$ and $10\hat{\imath}-\hat{\jmath}+6\hat{k}$ form a right angled triangle.

6. If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$, $\vec{b} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ and $\vec{c} = -3\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ find the magnitude and direction cosines of

   (i) $\vec{a} + \vec{b} + \vec{c}$ (ii) $3\vec{a} - 2\vec{b} + 5\vec{c}$

7. The position vectors of the vertices of a triangle are $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $-2\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$. Find the perimeter of the triangle.

8. The position vectors $\vec{a}, \vec{b}, \vec{c}$ of three points satisfy the relation $2\vec{a} - 7\vec{b} + 5\vec{c} = \vec{0}$. Are these points collinear?

9. Show that the points A (1, 1, 1), B(1, 2, 3) and C(2, - 1, 1) are vertices of an isosceles triangle.

10. Show that the points (4, - 3, 1), (2, - 4, 5) and (1, - 1, 0) form a right angled triangle.

11. If $|\vec{a}| = 5$, $|\vec{b}| = 6$, $|\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$.

12. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + 2\vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.

13. Show that the vectors $-\hat{\imath} - 2\hat{\jmath} - 6\hat{k}, 2\hat{\imath} - \hat{\jmath} + \hat{k}$, and $-\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ form a right angled triangle

14. Find the vectors of magnitude 6 which are perpendicular to both vectors $\vec{a} = 4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = -2\hat{\imath} + j - 2\hat{k}$.

15. Find the vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane which contains $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$.

16. Find the area of the parallelogram whose two adjacent sides are determined by the vectors

    $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$.

17. Find the area of the triangle whose vertices are A(3, - 1, 2), B(1, - 1, - 3) and C(4, - 3, 1).

III. <u>Answer the Following (5 marks)</u>

1. Theorem: Let O be the origin. Let A and B be two points. Let P be the point which divides the line segment AB internally in the ratio m : n. If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B, then the position vector $\overrightarrow{OP}$ of P is given by $\overrightarrow{OP} = \frac{n\vec{a} + m\vec{b}}{n+m}$.

2. Theorem: The medians of a triangle are concurrent.

3. Let A, B, and C be the vertices of a triangle. Let D,E, and F be the midpoints of the sides BC, CA, and AB respectively. Show that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \vec{0}$.

4. If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$.

5. Show that the vectors $5\hat{\imath} + 6\hat{\jmath} + 7\hat{k}, 7\hat{\imath} - 8\hat{\jmath} + 9\hat{k}, 3\hat{\imath} + 20\hat{\jmath} + 5\hat{k}$ are coplanar.

6. Show that the points whose position vectors $4\hat{\imath} + 5\hat{\jmath} + \hat{k}, -\hat{\jmath} - \hat{k}, 3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ are coplanar.

7. Show that the points (2, - 1, 3), (4, 3, 1) and (3, 1, 2) are collinear.

8. If $\vec{a},\vec{b}$ are unit vectors and θ is the angle between them, show that

   (i) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$ (ii) $\cos\frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$ (iii) $\tan\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$.

9. Let $\vec{a},\vec{b},\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

10. Three vectors $\vec{a},\vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 4$, and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Find $4\vec{a}\cdot\vec{b} + 3\vec{b}\cdot\vec{c} + 3\vec{c}\cdot\vec{a}$.

11. For any vector $\vec{a}$ prove that $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{k}|^2 + |\vec{a} \times \hat{k}|^2 = 2|\vec{a}|^2$.

12. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. Prove that $\vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b} \times \vec{c})$.

13. Find the angle between the vectors $2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + \hat{k}$ using vector product.