ST. JOSEPH STUDY CENTRE

12th STD: Quarterly Exam **MATHS** **Time: 2 Hrs/ Total Marks: 70**

I. Choose The correct answer: [15 x 1 = 15]

1. If $A = \begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$, then A =

   (1) $\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$ (3) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ (4) $\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$

2. If $A = \begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B = \begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then |adj (AB)| =

   (1) – 40 (2) -80 (3) -60 (4) -20

3. If $A^T A^{-1}$ is symmetric, then A^2 =

   (1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

4. If $A = \begin{bmatrix}\cos\theta & \sin\theta\\ -sin\theta & \cos\theta\end{bmatrix}$ and A(adj A) = $\begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, then k =

   (1) 0 (2) sin θ (3) cos θ (4) 1

5. The area of the triangle formed by the complex numbers z,iz, and z + iz in the Argand's diagram is

   (1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

6. If |z| = 1, then the value of $\frac{1+z}{1+\bar{z}}$ is

   (1) z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

7. A zero of $x^3 + 64$ is

   (1) 0 (2) 4 (3) 4i (4) – 4

8. If α, β , and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum\frac{1}{\alpha}$ is

   (1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

9. The value of $\sin^{-1}(\cos x)$, $0 \le x \le \pi$ is

   (1) π – x (2) $x - \frac{\pi}{2}$ (3) $\frac{\pi}{2} - x$ (4) x – π

10. The domain of the function defined by $f(x) = \sin^{-1}\frac{10}{\sqrt{x-1}}$ is

    (1) [1, 2] (2) [-1, 1] (3) [0, 1] (4) [-1, 0]

11. If the function $f(x) = \sin^{-1}(x^2 - 3)$, then x belongs to

    (1) [-1, 1] (2) $[\sqrt{2}, 2]$ (3) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (4) $[-2, -\sqrt{2}]$

12. The equation of the circle passing through (1,5) and (4,1) and touching y -axis is $x^2 + y^2 - 5x - 6y + 9 + \lambda(4x + 3y - 19) = 0$ where λ is equal to

    (1) $0, \frac{40}{9}$ (2) 0 (3) $\frac{40}{9}$ (4) $\frac{-40}{9}$

13. The centre of the circle inscribed in a square formed by the lines $x^2 - 8x - 12 = 0$ and $y^2 - 14y + 45 = 0$ is

    (1) (4,7) (2) (7, 4) (3) (9,4) (4) (4,9)

14. The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to the line $2x + 4y = 3$ is

    (1) x + 2y = 3 (2) x + 2y + 3 = 0 (3) 2x + 4y + 3 = 0 (4) x − 2y + 3 = 0

15. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0,3) is

    (1) $x^2 + y^2 - 6y - 7 = 0$ (2) $x^2 + y^2 - 6y + 7 = 0$ (3) $x^2 + y^2 - 6y - 5 = 0$ (4) $x^2 + y^2 - 6y + 5 = 0$

II. Answer any 10 of the following questions: [10 x 2 = 20]

16. If A = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$.
17. Find the rank of the following matrices by minor method: $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$
18. Evaluate the following if z = 5 - 2i and w = -1+ 3i $z^2 + 2zw + w^2$.
19. Find the modulus of the following complex numbers $\frac{2i}{3+4i}$.
20. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was left standing.
21. Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$ [if sum of two of its roots vanishes.
22. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.
23. Find the value of $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$.
24. Find the value of $\tan\left(cos^{-1}\left(\frac{1}{2}\right) - sin^{-1}\left(-\frac{1}{2}\right)\right)$.
25. Find the equation of circles that touch both the axes and pass through (−4,−2) in general form.
26. Find centre and radius of the following circles $2x^2 + 2y^2 - 6x + 4y + 2 = 0$.
27. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

III. Answer any 7 of the following questions: [7 x 5 = 35]

28. Solve the cubic equations : $8x^3 - 2x^2 - 7x + 3 = 0$.
29. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹ 150. The cost of the two dosai, two idlies and four vadais is ₹ 200. The cost of five dosai, four idlies and two vadais is ₹ 250. The family has ₹ 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had?
30. If $z_1 = 2 + 5i$, $z_2 = -3 - 4i$, and $z_3 = 1 + i$, find the additive and multiplicative inverse of z_1, z_2, and z_3.
31. Find the square roots of $-5 - 12i$.
32. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.
33. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.
34. Find the domain of the following $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$
35. Find the value of $cos^{-1}\left(cos\left(\frac{4\pi}{3}\right)\right) + cos^{-1}\left(cos\left(\frac{5\pi}{4}\right)\right)$.
36. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.
37. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
    $9x^2 - y^2 - 36x - 6y + 18 = 0$.
38. Solve the equations : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.